Influence of Optimised Sensor Placements on Multi-Objective Model Updating for Damage Localisation

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ABSTRACT

Effectively detecting and localising structural damage is crucial for ensuring the safety and longevity of engineering structures, making Structural Health Monitoring (SHM) an essential field of study. Vibration-based methods are often used to identify and localise occurring damage. These methods are based on monitoring the dynamic behaviour using recorded acceleration or strain data and detecting changes in this behaviour. For a more precise localisation, the observed changes are reproduced in a simulation model by means of optimisation to obtain more meaningful information. These model-based approaches are often applied in the context of digital twins. However, the position of the sensors used to record the applied data greatly impacts the localisation results. Several approaches for optimising sensor placements are available, yet their impact on the damage localising capabilities of model-based approaches is rarely analysed. This contribution uses a stochastic multi-objective optimisation approach to take parametric uncertainties into account in the sensor placement optimisation. Further, the effect of the optimised sensor layouts on the localisation results of model updating is investigated. The approach is applied to the Leibniz University structure for Monitoring (LUMO), a 9 m tall lattice tower with several reversible damage mechanisms. Modal properties are identified for different sensor setups with varying numbers of sensors. A stochastic multi-objective model updating procedure is applied using the identified modal properties to localise a damage scenarios in the benchmark structure under examination.
The results of the model updating procedure are compared for different optimal sensor setups, enhancing the insight into the impact of the sensor setup itself. The findings highlight the importance of strategic sensor placement in SHM applications, enabling more reliable model-based damage localisation.

Keywords: Optimal Sensor Placement, Model Updating, Multi-Objective Optimisation, Parametric Uncertainty

1. INTRODUCTION

Structural health monitoring (SHM) could play a decisive role in the safety of buildings in the future by enabling the precise detection and localisation of damage. Usually, the dynamic properties of a structure, e.g., in the form of modal parameters, are continuously identified and monitored. Changes in structural properties of the structure due to stiffness or mass changes, result in variations of the dynamic properties and thus can be detected by SHM-methods [1]. One possibility for damage localisation and quantification is model updating. Initial information of a structure is used to construct a numerical model. The difference between the dynamic properties of the structure and the model is quantified using an objective function. Subsequently, if the dynamic properties of the real structure change, optimisation procedures can be applied to minimise the objective function by adapting the numerical model, thus providing valuable information about potential damage scenarios [2].

However, the accuracy and reliability of the results obtained by the applied SHM-method heavily relies on the sensor setup, especially when mode shapes are also considered. Consequently, plenty of methods for solving the so-called optimal sensor placement (OSP) problem are available (cf. Tan and Zhang [3]). These methods can mainly be classified into sub-optimal methods and methods based on optimisation strategies [4]. While the sub-optimal methods iteratively change the sensor setup, e.g., by excluding sensors which do not contribute to the independence of the mode shapes, methods based on optimisation strategies treat the problem as a mathematical problem, where an optimised sensor setup is approximated by minimising objective functions. Various optimisation approaches are applied to minimise the optimisation problem, which can be formulated using different forms of objective functions [3], most of them by applying some form of model to assess the information given by a certain sensor setup. However, treating the problem as a deterministic problem and thus neglecting the parametric uncertainty of the simulation model, e.g., uncertain Young's modulus, can lead to misleading or insufficient results. Castro-Triguero et al. [4] showed that the parametric uncertainties can have a significant impact on the optimal sensor configuration obtained by the approach. Further, as different objective functions also come with their own benefits and disadvantages, the use of multi-objective optimisation approaches has increased (cf. Lin et al. [5]). In this work, we aim to incorporate multiple objective functions and parametric uncertainties to combine their merits. To this end, a stochastic multi-objective optimisation approach is used to determine sensor setups with a pre-defined maximum number of senors on the Leibniz University Test structure for monitoring (LUMO), which is a 9 m tall, outdoor girder mast structure with several reversible damage mechanisms. Five different sensor setups with different number of sensors are determined. The modal properties of the structure are identified using the Bayesian operational modal analysis method (BAYOMA) for different wind speeds, enabling the comparison of the sensor setups across multiple signal to noise ratios. The potential damage is located via model updating, incorporating the same stochastic multi-objective optimisation approach.

2. OPTIMAL SENSOR PLACEMENT

2.1. Stochastic Multi-Objective Optimisation

This study used the recently introduced stochastic multi-objective pattern search (SMOPS) (cf. Ragnitz et al. [6]) approach. This approach is based on the multi-objective pattern search (MOGPS), described by Günther et al. [7]. It employs a deterministic, integer grid-based sampling pattern to generate new parameter values within the n-dimensional search space $\mathcal{G} \in \mathbb{N}^n$ (cf. Figure 1). The samples are then projected onto a floating grid in the design space $\mathcal{X} \in \mathbb{R}^n$ within the bounds $[\boldsymbol{x}_{\text{lb}}, \boldsymbol{x}_{\text{ub}}]$. It maintains a collection of tracked samples, referred to as the *hall of fame*, which contains at least T samples representing various levels of Pareto fronts. New samples are created by applying the grid-based sampling pattern to the tracked samples in the hall of fame. If there is no change in the hall of fame members over two successive iterations, one of the base vectors w_i , is reduced in length, and the process is repeated till a predetermined number of samples N_{eval} is reached.

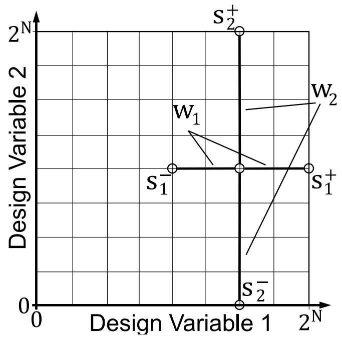


Figure 1: Integer grid based sampling approach of the MOGPS [7].

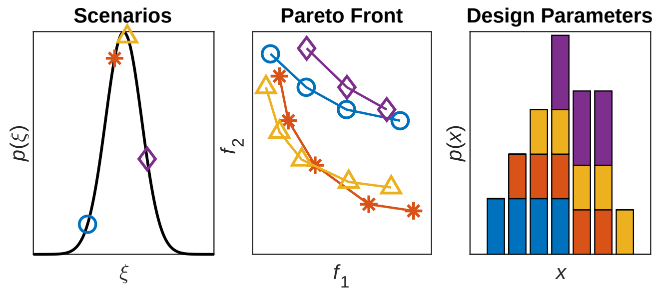


Figure 2: Basic idea to estimate the probability p for a design variable x with several scenarios ξ in the context of multi-objective optimisation for two objectives f_1 and f_2 [6].

If an uncertain parameter is supposed to be included in the optimisation approach itself the SMOPS approach can be applied. Uncertain parameters can be included by generating so-called scenarios corresponding their distribution. Every design parameter that is Pareto-efficient for at least one of the generated scenarios is included in the hall of fame. The objective function is evaluated for every scenario and design parameter individually. The basic principle of the optimisation procedure remains equivalent to the deterministic MOGPS. As each scenario leads to different objective values for the same design parameters $\boldsymbol{x}$, the result can be interpreted as individual Pareto-fronts for every scenario considered (cf. Figure 2). Due to the deterministic search pattern of the approach it is expected that a design parameter can be Pareto-efficient for multiple scenarios. Therefore, each Pareto-optimal design vector $\boldsymbol{x}$ is weighted based on the number of estimated Pareto-optimal solutions and the total number of scenarios.

2.2. Senor Setup Optimisation

The optimal sensor placement problem can be treated as an optimisation problem with the objective of finding a sensor setup based on the design vector $\boldsymbol{x}$, which best reduces one or several objective functions. The design vector $\boldsymbol{x}$ itself depends on the problem. It could be used to assign a sensor to a specific position based on the coordinates of a three-dimensional structure or to assign sensors to predefined candidate degrees of freedom (DOFs) suitable for potential sensor locations, e.g., by means of combinational optimisation. Since the basis of the SMOPS is an integer grid, the algorithm is well suited for the assignment of sensors to discrete degrees of freedom, as well as continuous sensor placements. In this study, the approach is used to assign sensors to discrete positions. A design vector $\boldsymbol{x}$ is utilised, which contains the index for every sensor, i.e., its size corresponds to the maximum number of sensors. The upper boundary of the integer grid is restrained according to the number of potential sensor positions. An additional sensor position can be introduced, which ignores the impact of the sensor, allowing the optimisation to also consider a lower number of sensors (cf. Section 3.2.).

Parameter uncertainty, that stems from a lack of knowledge about the properties of a physical system, influences the output of sensor optimisation, i.e., the parameters to be used for the simulation model are not know precisely. As shown by Castro Triguero et al. [4], parameter uncertainty can be incorporated using Monte Carlo samples, or scenarios, generated corresponding to the probability distribution of the parameter susceptible to parameter uncertainty. Here, this problem is tackled with the SMOPS presented in Section 2.1.. The optimisation problem is solved for each generated scenario.

Two frequently used objective functions are utilised [8]. The first one is based on maximising the determinant of the fisher information matrix, while minimising the condition number and thereby the ill-posedness of the problem (cf. Yan and Cia [8]). This is facilitated by the objective function

$$\varepsilon_{\mathrm{FI}} = \sqrt{\lambda_1/\lambda_{N_{\mathrm{Modes}}}} \prod_{i=1}^{N_{\mathrm{Modes}}} \lambda_i, \tag{1}$$

where λ_i is the i-th eigenvalue of the matrix $\mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi}$, with $\mathbf{\Phi}$ being the mode shape matrix. A second objective function is applied, based on minimising the modal assurance criterion (MAC) value between different mode shapes and thus the correlation between mode shapes Φ_i and Φ_j when using a certain sensor setup (cf. Yan and Cia [8]). To account for the possibility of closely space modes, we adapt this formulation slightly by substituting the subspace of order 2 modal assurance criterion (S2MAC) presented by D'Ambrogio and Fregolent [9] for the MAC value. It quantifies the correlation between a mode shape Φ_i and a mode shape subspace spanned by two mode shapes $[\Phi_j, \Phi_k]$

$$\mathrm{S2MAC}_{i,jk} = \max_{\alpha,\beta} \frac{|\Phi_i^*(\alpha\Phi_j + \beta\Phi_k|^2}{||\Phi_i||_2^2 \, ||\alpha\Phi_j + \beta\Phi_k||_2^2}. \tag{2}$$

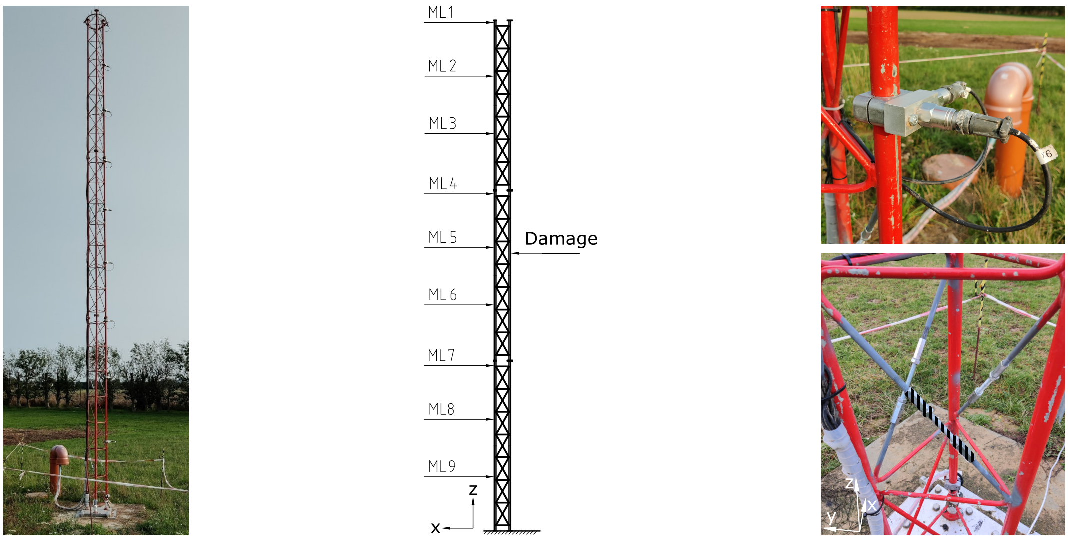
$(\,)^*$ indicates the conjugate transpose. The objective is to minimise the correlation, thus minimising the maximum value in the off-diagonal terms of the S2MAC matrix.

$$\varepsilon_{\mathrm{S2MAC}} = \max_{i \neq j,k} \mathrm{S2MAC}_{i,jk} \quad \text{for} \quad i,j,k \in [1, N_{\mathrm{Modes}}] \tag{3}$$

3. APPLICATION

3.1. LUMO

To validate the approach, the Leibniz University Structure for Monitoring (LUMO) data set is employed, which was collected from the 9-meter-tall lattice structure depicted in Figure 3a. Additional details about the experimental setup and the structure are available in Wernitz et al. [10]. The existing measurement setup consists of nine measurement levels (ML), each equipped with two acceleration sensors (cf. Figure 3c). Structural damage can be incorporated by removing a diagonal struts, as depicted in Figure 3c and Figure 3b. Only one damage position is considered in this work, as it poses the biggest challenge of the existing damage cases for damage localisation (cf. Ragnitz et al. [6]). The corresponding numerical



(a) Photograph of girder mast structure.

(b) Measurement levels (ML) and damage positions.

(c) Sensors at ML9 (top) and a reversible damage mechanism with a marked damage position. (bottom)

Figure 3: Leibniz University Structure for Monitoring [10].

model is assembled in Abaqus and consists of three-dimensional Timoshenko beams. As the structure under examination is an outdoor structure, varying temperatures and wind speeds are inevitable, which will impact the results of the SHM procedure itself. However, as varying environmental conditions are not the focus of this contribution, we chose data sets with similar conditions for the comparison.

Therefore, all data sets under examination were recorded with an outdoor temperature close to $17°$ C. Further, we chose data sets with three different wind speeds (approximated at $2,\ 5.5\,\&\,8.5\frac{\mathrm{m}}{\mathrm{s}}$), as the wind speed acts as the primary excitation of the structure. For each state of the environmental conditions, ten data sets are used, resulting in 30 data sets.

3.2. Sensor Setups

The optimisation approach, presented in Section 2.2., is applied to the LUMO test structure. As the data has been recorded with the measurement setup depicted in Figure 3, the candidate sensor positions in the optimisation are restricted to those available, considering the x- and y acceleration sensors separately, i.e., 18 candidate positions. Similar to the approach presented in Castro-Triguero et al. [4], three different model parameters are chosen as uncertain parameters, namely the cross-sectional area, Young's modulus and the mass density of all beams. The coefficient of variation is set to 5% for all of the above. The resolution of the integer grid is set to 19 to enable all sensor positions, as well as one sensor position, which leads to the sensor not having an impact on the result. In total, three different optimisation runs with varying numbers of maximum sensors (4,9,12) are conducted. Figure 4 depicts the results of these optimisation procedures for the three most probable sensor setups. For a maximum number of four

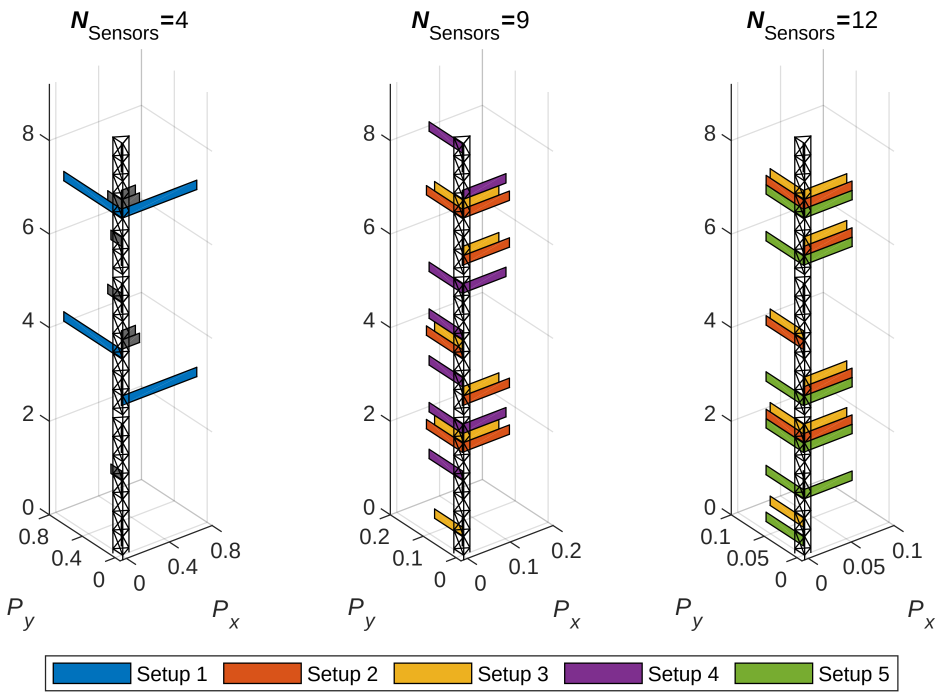


Figure 4: The probability P for the three most probable sensor setups estimated with the sensor optimisation procedure for a limited maximum number of sensors N_{Sensor}.

sensors ($N_{\mathrm{Sensor}} = 4$), the setup depicted in blue is estimated with a probability of $P = 78\%$ (Setup 1). Since the remaining optimal sensor setups only make up a small portion, only the most probable solution is considered further in the model updating step. With a maximum number of sensors equal to nine ($N_{\mathrm{Sensor}} = 9$), three different setups are estimated with a similar probability. All three setups differ in the sensor positions and the estimated number of sensors. Therefore, all three setups are considered in the model updating step. Finally, the procedure is run with a maximum number of 12 sensors ($N_{\mathrm{Sensor}} = 12$). Again, the three most probable solutions are shown. The solution estimated with the highest probability possesses eleven different sensor positions (Setup 5). The remaining two sensor setups are two of the solutions also found when running the optimisation with a maximum number of nine sensors. In total, five different sensor setups with varying sensor numbers are considered in the model updating step.

3.3. Model Update

The different sensor setups are now applied to a typical model updating procedure. All sensor setups are used to identify the eigenfrequencies and mode shapes of the first two bending mode pairs, as well as their standard deviations and covariance, respectively. The identification is carried out with the BAYOMA method employing 10 min long time intervals.

The model updating procedure to locate and quantify the damage used here was first presented in Ragnitz et al. [2] for the deterministic case and later expanded by Ragnitz et al. [6] to incorporate uncertainties into the multi-objective optimisation approach. The modal properties are compared based on two objective functions. One comparing the relative decrease in eigenfrequencies between the healthy $f_{\mathrm{S0},k}$ and damaged $f_{\mathrm{S1},k}$ state of the numerical model with the identified eigenfrequencies for the healthy $f_{\mathrm{M0},k}$ and damaged state $f_{\mathrm{M1},k}$ of the real structure. The second objective function compares the mode shapes $\boldsymbol{\Phi}$ accordingly.

$$
\begin{aligned}
\varepsilon_f^2(\boldsymbol{x}) &= \sum_{k=1}^{N_{\mathrm{modes}}} \left(\frac{f_{\mathrm{S1},k}(\boldsymbol{x}) - f_{\mathrm{S0},k}}{f_{\mathrm{S0},k}} - \frac{f_{\mathrm{M1},k} - f_{\mathrm{M0},k}}{f_{\mathrm{M0},k}} \right)^2 \\
\varepsilon_\Phi^2(\boldsymbol{x}) &= \sum_{k=1}^{N_{\mathrm{modes}}} ||(\Phi_{\mathrm{S1},k}(\boldsymbol{x}) - \Phi_{\mathrm{S0},k}) - (\Phi_{\mathrm{M1},k} - \Phi_{\mathrm{M0},k})||_2^2 \,.
\end{aligned} \tag{4}
$$

To incorporate damage into the numerical model, a rather simple damage parameterisation based on two damage parameters is employed. The stiffness of a defined beam in the model i_{Beam} is reduced by a factor D, while a factor $D = 1$ corresponds to a complete removal of the beam. A separate optimisation procedure is run for every data set and sensor setup available. All optimisation procedures are stopped after a total of 3000 model evaluations. Uncertainty present in the eigenfrequencies and mode shapes is again incorporated by generating scenarios based on their respective probability distributions. A detailed discussion can be found in Ragnitz et al. [6]. The results are illustrated in Figure 5 via different bar plots

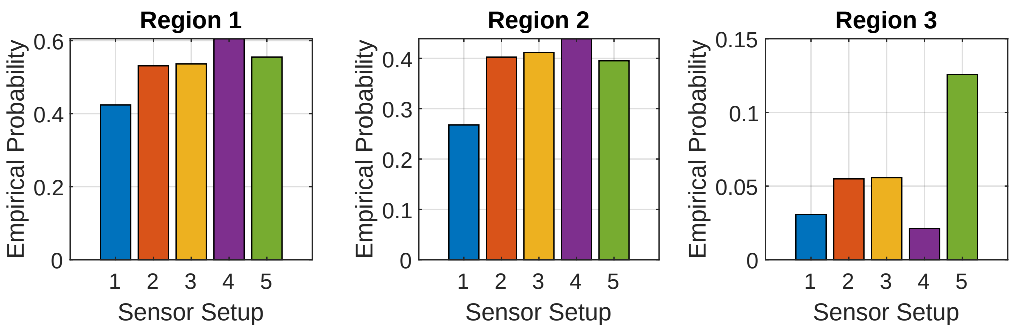


Figure 5: Empirical probability of localising the correct region for all five Sensor Setups.

visualising the probability of damage in different regions. The results are averaged over all thirty data sets under examination. Region 1 indicates that the damage is located in the correct hub or the hub directly above or below the correct one. Region 2 indicates the probability of the damage in the correct hub, while Region 3 indicates that the correct beam is localised. All sensor setups localise the damage with some probability in the correct region. The results for regions one and two are similar to each other, i.e., sensor setups with more than four sensors predict damage in the region with a similar percentage, while the first sensor setup's result is less accurate. For the localisation of the correct beam (Region 3), sensor setup five shows a significant improvement in the results compared to the other setups. The optimal sensor setup for a maximum number of 12 sensors (Setup 5) arguably shows superior performance in comparison to the remaining setups estimated for the respective maximum number of sensors. The results of sensor setups two and three are almost equal across all data sets, as they only differ in one sensor position.

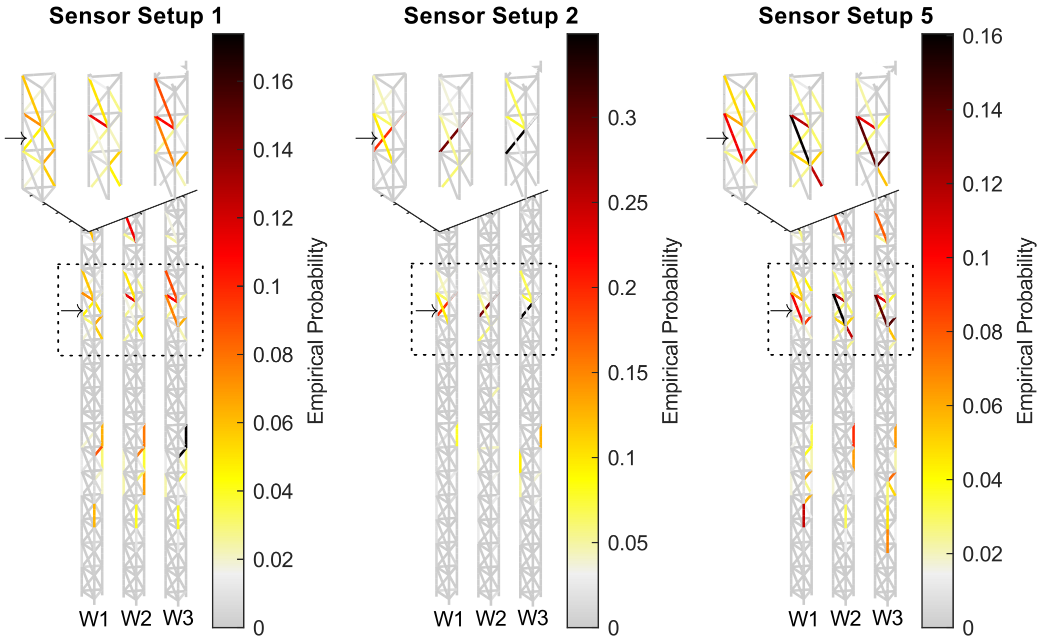


Figure 6: Localisation results for sensor setups one, two and five depicted on the actual lattice tower for three different wind speeds W1, W2 and W3. The arrow marks the true damage position

The results for setups one, two and five are now depicted on the actual lattice tower for three different wind speeds $(2,\ 5.5\ \&\ 8.5\frac{\text{m}}{\text{s}})$ (cf. Figure 6). The results are averaged over the ten data sets for each wind speed. The empirical probability of the localisation for the correct area generally increases with increasing wind speeds, as higher windspeeds also result in higher amplitudes and, therefore, higher signal-to-noise ratios. The localisation accuracy increases with a higher number of sensors. While for setup one, the beams in close proximity to the actual damage position only make up a small portion of the results, this drastically increases for setup two. The beam predicted with the highest probability is a diagonal beam at the correct height but with a wrong orientation. However, for setup five, the model updating accomplishes predicting the correct beam with the highest percentage for wind speeds two and three.

4. CONCLUSION

This work presented a stochastic multi-objective approach for the optimisation of sensor setups under parametric uncertainty. The presented approach was applied to an outdoor benchmark structure and used to estimate five different sensor setups for three different maximum numbers of sensors. All different setups were applied to a damage localisation procedure via model updating. A successful localisation of the damage was only possible when using the setup with the highest number of sensor considered. Optimal sensor setups were shown to enhance the detection capabilities, with increased numbers of sensors leading to higher probabilities of correctly identifying both the region and specific location of the damage. This study not only affirms the benefits of optimised sensor deployment in SHM but also advocates for the integration of advanced optimisation techniques to accommodate uncertainties, thereby providing a robust framework for real-world SHM applications. While this work tried to estimate optimal sensor setups with a given number of maximum sensors, the question regarding the needed number of sensors was not considered. Further, it could be beneficial to take possible damage scenarios into account when optimising a certain sensor setup. Thereby, specifications for the sensor setups could be included and lead to a more meaningful results.

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