



International Operational Modal Analysis Conference

20 - 23 May 2025 | Rennes, France

Long-term dynamic monitoring of the Leaning Tower of Pisa

Laura Vignali¹, Anna De Falco², Kristof Maes³, Edwin Reynders⁴

¹ University of Pisa, Department of Civil and Industrial Engineering, Largo Lucio Lazzarino 1, Pisa - Italy, laura.vignali@phd.unipi.it

² University of Pisa, Department of Civil and Industrial Engineering, Largo Lucio Lazzarino 1, Pisa - Italy, anna.de.falco@unipi.it

³ KU Leuven, Department of Civil Engineering, Structural Mechanics Section, Kasteelpark Arenberg 40, 3001 Leuven - Belgium, kristof.maes@kuleuven.be

⁴ KU Leuven, Department of Civil Engineering, Structural Mechanics Section, Kasteelpark Arenberg 40, 3001 Leuven - Belgium, edwin.reynders@kuleuven.be

ABSTRACT

Dynamic Structural Health Monitoring, as a non-destructive method, plays a crucial role in the conservation of cultural heritage structures. Specifically, long-term monitoring enables continuous tracking of a structure's dynamic behavior, contributing to a deeper understanding of its responses to environmental factors and the progression of its condition over time.

This paper focuses on the modal identification of the Leaning Tower of Pisa. In the past, the Tower has undergone several dynamic identification campaigns; however, each was conducted sporadically and for only short durations. In July 2023, for the first time, a continuous dynamic acquisition system was installed on the Tower, featuring 21 channels dedicated to long-term monitoring of its acceleration response.

The collected data were processed using the Covariance-driven Stochastic Subspace Identification (SSI-cov) algorithm, implemented via MACEC Matlab toolbox. This approach enabled the extraction of the Tower's natural frequencies, vibration modes, and damping ratios, as well as the tracking of their fluctuations over time. The results were then compared with findings from previous dynamic campaigns and modal data from a Finite Element Model of the Tower.

This research advances knowledge of the Tower's response to environmental influences and provides insights into soil-structure interaction, establishing a solid foundation for enhancing future structural health assessments of this and similar heritage assets.

Keywords: Operational Modal Analysis, Structural Health Monitoring, Stochastic Subspace Identification method, Leaning Tower of Pisa

1. INTRODUCTION

The Leaning Tower of Pisa has experienced a progressive tilt since its construction due to the subsoil's high deformability. When the inclination reached 5.5° southward, concerns arose about its stability. Stabilization efforts, including controlled soil extraction in 2000, reduced the tilt by 0.5° , restoring it to its 19th-century state. Though the tilt has decreased, continuous structural health monitoring (SHM) and further studies remain crucial to ensuring its long-term stability.

Extensive geotechnical investigations have thoroughly characterized the subsoil [1]; however, the mechanical properties of the masonry remain less explored. In this regard, the recently installed continuous dynamic monitoring system—the first of its kind for the Tower—offers a unique opportunity to refine masonry property estimates and assess the influence of environmental factors on its dynamic behavior, enabling a more accurate evaluation of the Tower's long-term structural condition.

This study marks an initial step in modal identification, assessing whether the current monitoring system is adequate for long-term evaluations or requires improvements, such as optimizing sensor number and placement. In addition, these investigations lay the foundation for defining alert thresholds and enhancing real-time monitoring strategies to support the conservation and structural health of the monument.

2. SHM SYSTEM OF THE LEANING TOWER AND PREVIOUS DYNAMIC CAMPAIGNS

The Leaning Tower of Pisa has been monitored since the early 20th century, with significant advancements in its SHM system occurring during the stabilization works of the 1990s. At that time, numerous sensors were installed to assess its condition. Following these interventions, the SHM system evolved to support long-term monitoring. Today, the Tower is equipped with deformometers, thermometers, pendulums, electro-levels, and a weather station measuring air temperature, solar radiation, air pressure, and wind speed. Additionally, four piezometers near the foundation monitor the water table level and subsurface temperature.

The Tower was also the focus of some dynamic monitoring campaigns [2], consistently revealing that the first two bending modes occur around 1 Hz. However, only a few studies identified a third mode, which is likely extensional and appears near 3 Hz. Peaks around 6 Hz were variably attributed to second-order bending or torsional modes. Table 1 summarizes the findings of the most recent campaign, based on data from four accelerometers installed in the Tower in 2002 and triggered by seismic events [3].

Table 1: Natural frequencies of the Leaning Tower identified by Fiorentino et al. [3].

Mode	Type	Frequency [Hz]
1	Bending N-S	0.96
2	Bending E-W	1.02
3	Extensional	2.97
4	Torsional?	6.29

In this study, we analyzed data from the first continuous dynamic monitoring system of the Tower, installed in July 2023 as part of a project funded by the Italian Ministry of Culture (MiC). The system consists of five triaxial and three biaxial accelerometers, totaling 21 measurement channels distributed across different heights of the structure (Figure 1a). Data were initially recorded at 500 Hz until March 2024, when the sampling rate was reduced to 100 Hz.

3. FE MODEL OF THE LEANING TOWER AND MODAL ANALYSIS

A Finite Element Model (FEM) of the Leaning Tower was previously developed in COMSOL Multiphysics [4] using a highly detailed point cloud [5]. The model (Figure 1b), consisting of 258850 tetrahedral elements, accurately represents the Tower's inclination, its curvilinear shape, the presence of an

internal staircase, and the layered wall structure, which comprises an external marble layer and a mixed masonry infill. The underlying soil was also modeled (Figure 1c), consisting of four distinct layers identified in the literature [1] and extending to a depth of 100 m. The other two dimensions (360 m) were selected to minimize boundary conditions effect on the modal analysis. Material properties were assigned based on previous experimental tests conducted on both masonry layers (flat jack and dilatometer tests) and soil (cross-hole tests). A detailed description of the model is available in [5], where the impact of material uncertainties on the Tower's natural frequencies was assessed within a probabilistic framework. In this study, a deterministic model with mean material property values was used to compare numerical results with those from Operational Modal Analysis (OMA). The outcomes of the modal analysis are summarized in Table 2.

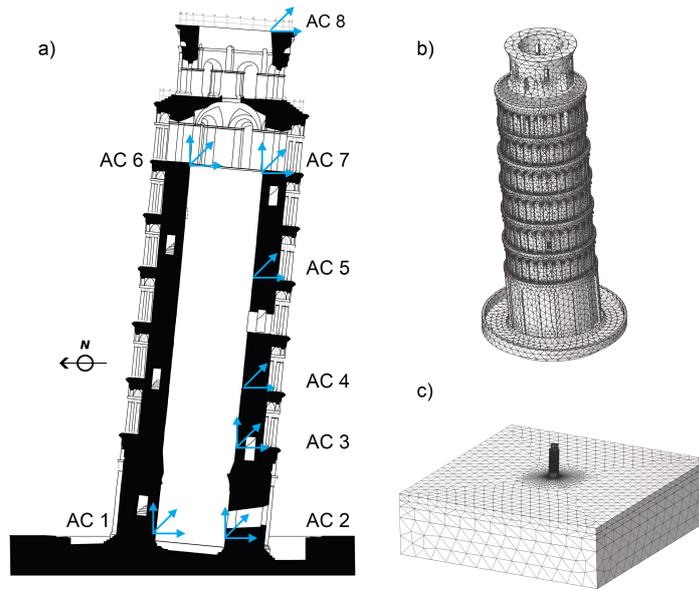


Figure 1: a) N-S section of the Tower and the current location of the accelerometer network; b) FEM of the Tower; c) FEM of the Tower with soil.

Table 2: Frequencies of the Leaning Tower obtained from the FEM.

Mode	Type	Frequency [Hz]	Mode	Type	Frequency [Hz]
1	Bending E-W	0.98	8	II Bending E-W	10.30
2	Bending N-S	0.98	9	II Torsional	13.40
3	Extensional	3.50	10	III Bending	18.19
4	Foundation movement	4.68	11	III Bending	18.29
5	Foundation movement	4.69	12	III Torsional	18.60
6	Torsional	5.78	13	Bell chamber	23.05
7	II Bending N-S	10.30	14	Bell chamber	23.10

4. DYNAMIC IDENTIFICATION

The authors analyzed one year of recordings (July 2023–July 2024) collected by the Tower's continuous monitoring system. Unfortunately, acceleration data from October 29 to December 14, 2023 is missing due to a storm that temporarily rendered the system inoperative. Additionally, damaged sensors AC 4, 5, and 7 (see Figure 1a) were excluded from the analysis until May 11, 2024.

Each one-hour recording was used to extract the Tower's dynamic characteristics, including frequency, damping, and mode shapes. Modal identification was performed using the MACEC MATLAB toolbox [6]. During preprocessing, the static component was removed, followed by high-pass filtering with a 0.1

Hz cut-off frequency and resampling at 50 Hz. The reference-based Covariance-driven Stochastic Subspace Identification (SSI-cov/ref) method [7] was then applied to generate a series of state-space models and stabilization diagrams. These diagrams, combined with various validation criteria [8], enabled the differentiation between physical and spurious modes. While manual selection of physical modes is feasible for a limited number of modal tests, automated selection processes are crucial for long-term SHM.

The automated procedure followed these steps: (1) *Target Modes Selection*: modes manually identified from the first recording served as targets for subsequent datasets, after verifying through a subset of recordings that no additional stable modes emerged; (2) *Candidate Modes Identification*: for each target mode, candidate modes were selected based on frequency proximity and validated using stabilization criteria to distinguish physical modes. The validation process also included computing the Modal Assurance Criterion (MAC) against the target mode shape, after removing outliers; (3) *Final Modes Selection*: the final modes for each recording were determined by minimizing deviations from the mean frequency of the candidate modes.

Only a subset of the target modes was consistently detected throughout the analyzed period. The main findings—including the mean, minimum, and maximum frequency values, their standard deviations, and the mean damping values for each mode—are summarized in Table 3. The first three modes of the Tower correspond to two bending modes along the North-South (N-S) and East-West (E-W) directions (around 1 Hz) and an extensional mode at approximately 2.5 Hz, in agreement with previous findings (Table 1). The fourth mode, observed around 6 Hz, represents a second-order bending mode along the N-S direction. The fifth and sixth modes primarily involve deformation concentrated in the upper part of the Tower, the bell chamber.

Initially, a potential third-order bending mode along the N-S direction was identified at 11.80 Hz. However, as it appeared in only a few recordings, it was excluded from the final analysis.

Figures 2 and 3 illustrate the fluctuations in frequency and damping of the identified modes throughout the year. The gray-shaded area indicates the period when seven channels were nonfunctional. During this time, a greater dispersion of results is evident, particularly for modes 4, 5, and 6, probably due to challenges in identifying higher modes without some data. A more comprehensive analysis of these modes during the winter and spring seasons should be conducted once new measurements become available. The notably high damping values observed for mode 4, and especially mode 3, can be attributed to the strong influence of soil-structure interaction on the Tower. This effect is particularly pronounced in relatively stiff structures built on soft soils, where energy dissipation occurs through wave propagation away from the structure, increasing the overall damping of the dynamic system [9]. This phenomenon, commonly known as radiation damping, has been reported in the literature with values reaching up to 30% [10]. As noted in [9], this effect appears to be less significant for rocking motions, which predominantly characterize the first two modes, as reflected in their lower damping values.

Table 3: Identified modes, including the mean, standard deviation, minimum, and maximum values of the frequencies, the mean damping values, and the MAC values computed against the corresponding numerical mode shapes.

Mode	Type	Mean freq. [Hz]	Min. freq. [Hz]	Max. freq. [Hz]	Sd. freq. [Hz]	Mean damp. [%]	MAC (FEM)
1	Bending N-S	1.03	1.02	1.04	0.003	2	0.92
2	Bending E-W	1.06	1.05	1.07	0.003	2	0.99
3	Extensional	2.58	2.36	2.80	0.08	24	0.57
4	II Bending N-S	6.00	5.70	6.26	0.09	16	0.61
5	Bell chamber	15.52	15.06	15.98	0.17	0.8	0.99
6	Bell chamber	17.14	16.72	18.06	0.29	0.8	0.93

The comparison between OMA results and FEM predictions showed good agreement in terms of MAC values for most modes (see Table 3), except for the extensional and second-order bending modes. The extensional mode is influenced by the absence of vertical components in three accelerometers; however, the numerical and experimental mode shapes are consistent. For the fourth mode, although visually

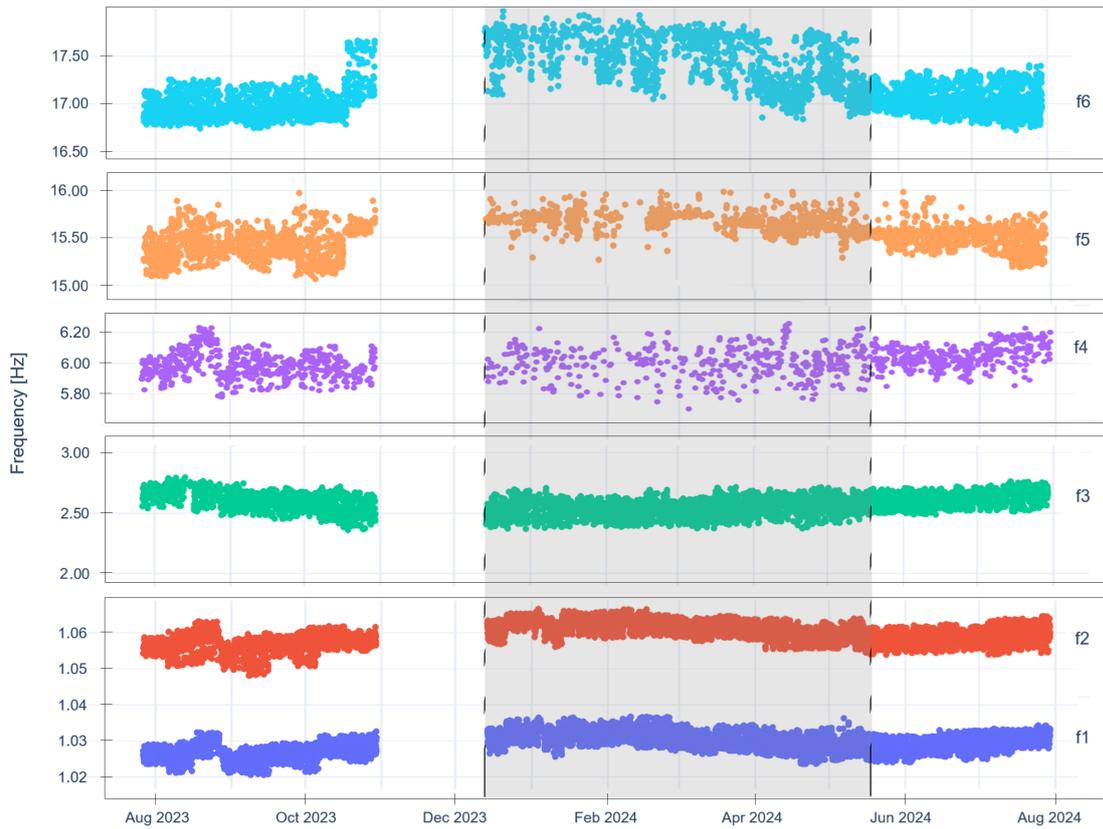


Figure 2: Identified natural frequencies of the Leaning Tower over one year of recordings. The grey-shaded area corresponds to the period when sensors AC 4, 5, and 7 were damaged and excluded from the analysis.

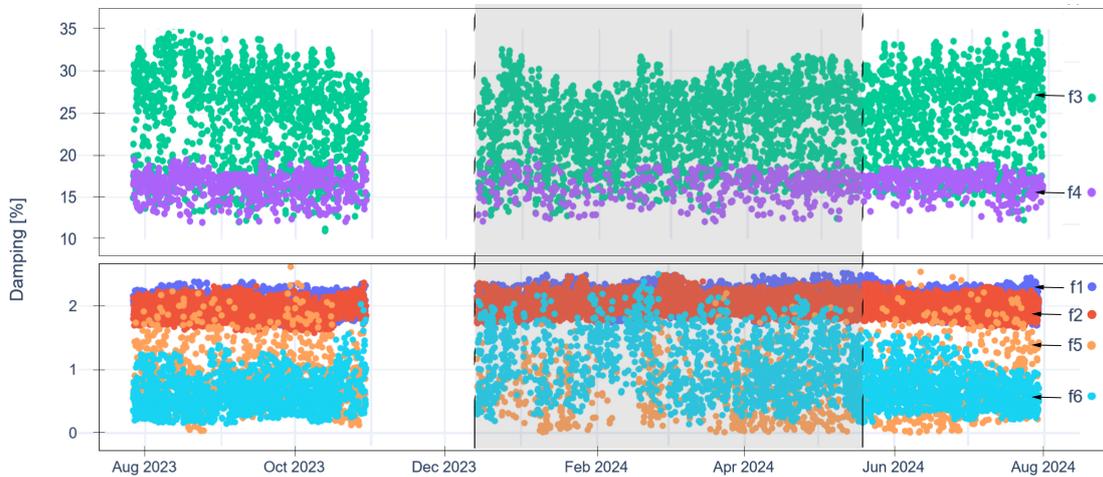


Figure 3: Damping associated to the natural frequencies of the Tower over one year of recordings. The grey-shaded area corresponds to the period when sensors AC 4, 5, and 7 were damaged and excluded from the analysis.

similar to its numerical counterpart, it exhibits a slightly different curvature, which likely explains the lower MAC value. Figure 4 illustrates the experimental mode shapes superimposed on the numerical ones.

Both methods identified the first two natural frequencies at approximately 1 Hz. However, in the FEM results, the E-W bending mode preceded the N-S mode, with a negligible frequency difference. In contrast, the experimental results showed a frequency gap of about 0.03 Hz, with the N-S mode occurring first.

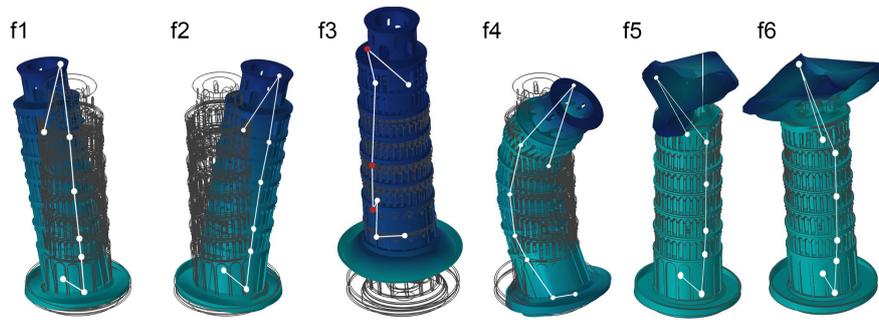


Figure 4: Overlay of numerical and experimental mode shapes of the Leaning Tower. Red dots in the extensional mode (f3) indicate sensors without a vertical channel.

The extensional mode was identified as the third mode in both the numerical model and experimental observations, though its frequency was slightly higher in the FEM. For higher modes, the FEM significantly overestimated the frequencies compared to the experimental values. This discrepancy is consistent with the sensitivity analysis reported in [5], which highlighted that second-order and higher modes are particularly sensitive to masonry properties, whose reliability is uncertain due to the limited number of tests, which were conducted without standardized procedures. Notably, the average Young's modulus assigned to the external masonry layer, based on available tests, was 50000 MPa—a value considered unrealistically high, even for well-crafted masonry.

These findings are essential for refining future model parameter identification, serving as a foundation for updating the assigned values of both masonry and soil properties.

5. INFLUENCE OF ENVIRONMENTAL PARAMETERS ON FREQUENCIES

Understanding the influence of environmental conditions on the identified frequencies is essential for distinguishing variations potentially driven by future damage from those caused by environmental factors. To investigate this, the frequency values were compared with the available static monitoring data, described in Section 2.

Figure 5 presents the six identified frequencies plotted against air temperature, a parameter widely reported in the literature as influencing the modal characteristics of masonry towers. Specifically, a rise in temperature typically leads to a linear increase in frequency values, particularly in the lower modes [11]. However, in this study, such behavior is clearly observed only for the third frequency and, to some extent, for the fourth one.

To further investigate this behavior, an in-depth analysis of the first two frequencies was conducted by examining their correlation with temperature variations throughout the year. This analysis revealed that while the correlation between frequencies and temperature was not always consistent, they aligned during certain periods, suggesting the influence of an additional variable. As shown in Figure 6, subsurface temperature and frequencies tend to increase together, indicating a correlation. Figure 7 provides valuable insights for investigating the interaction between these three involved variables. Specifically, Figure 7a examines the relationship between air and subsurface temperatures, revealing that when air temperature drops below 15°C, the soil tends to reach higher temperatures, whereas above 15°C, the subsurface temperature decreases, fluctuating around two distinct values. This finding confirms that f1 and f2 exhibit a classic linear dependency on air temperature; however, this trend becomes evident only when subsurface temperature is also considered (Figure 7b). In particular, this relationship leads to the formation of three distinct clusters in Figure 7b: the two on the right correspond to the summer season, while the one on the left represents winter. The most significant shift occurs around 15°C, a threshold that, as mentioned, appears to be critical for variations in subsurface temperature (Figure 7a), as well as for the water table level, suggesting that soil conditions may be the primary cause in frequency fluctuations. This seasonal

behaviour is consistent with previous findings reported in [11], although that study identified a different influence on frequencies.

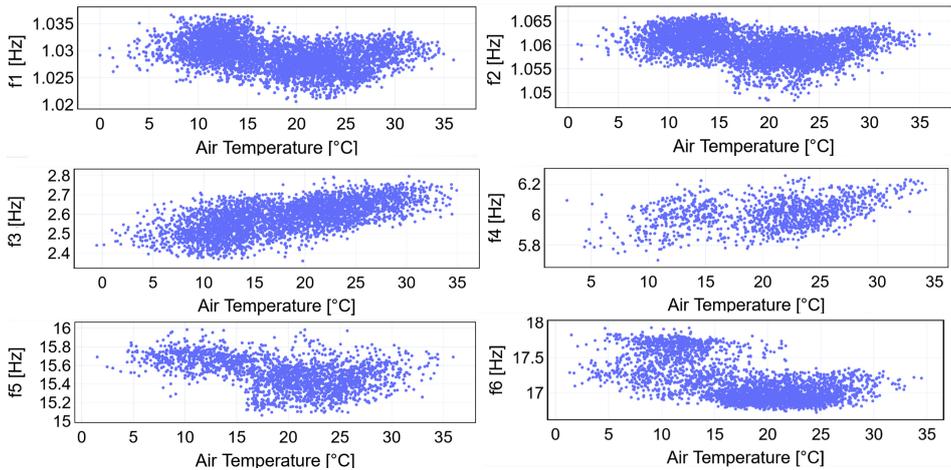


Figure 5: Frequencies of the identified six modes vs. air temperature.

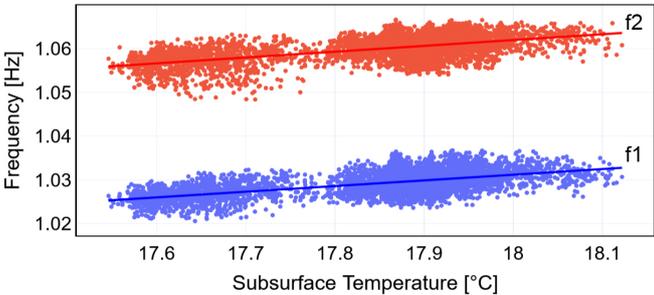


Figure 6: First two frequencies vs. soil temperature.

A linear relationship was also observed between the base tilt in the N-S direction and both f1 and f2. However, since this tilt is strongly correlated with subsurface temperature, it suggests that soil influences both the static and dynamic behavior of the system.

For higher frequencies, the results highlight the need for additional sensors at the top of the structure, as the current setup includes only a single biaxial accelerometer in the bell chamber. In particular, for mode 6, it appears that two distinct modes may have been grouped together, leading to two clusters (Figure 5) that, in this case, cannot be justified.

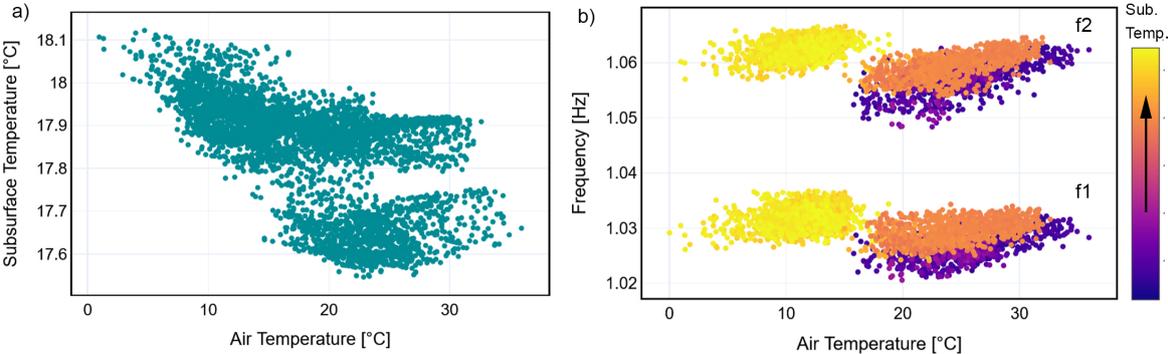


Figure 7: (a) Air temperature vs. soil temperature; (b) First two frequencies vs. air temperature, with marker colors representing the soil temperature values.

6. CONCLUSIONS

The analysis of data from the first continuous dynamic monitoring system enabled the identification of a larger number of Tower modes, which were characterized in terms of frequency, damping, and mode shape. A good agreement was found with the first modes previously identified in the literature and the results from the FEM, with particularly strong consistency in mode shapes for the latter. Observing these modes over the course of a year will provide an opportunity to use the results for updating masonry and soil parameters.

Furthermore, this analysis lays the foundation for understanding the influence of environmental variables, such as air and subsurface temperature, which require further investigation to assess their impact on dynamic behavior. However, the results, particularly for higher-order modes, need to be validated by analyzing data over a longer period, considering also the system failure that occurred during the monitoring period.

To conclude, this study represents an important preliminary step towards developing a real-time monitoring system capable of distinguishing between natural variations and those caused by potential future structural damage.

ACKNOWLEDGMENTS

Opera della Primaziale Pisana is gratefully acknowledged for providing the monitoring data. Grateful acknowledgment is extended to the Italian Ministry of Culture (MiC) for its support.

REFERENCES

- [1] D. Lo Presti, M Jamiolkowski, and M Pepe. Geotechnical characterisation of the subsoil of Pisa Tower. *haracterization and Engineering Properties of Natural Soils-Tan et al. (eds.)*, 1, 2003.
- [2] Dionis Butuc, Anna De Falco, Filippo Landi, et al. Bayesian updating of a Finite Element Model of the Leaning Tower of Pisa. In *Proc. of 20th International Probabilistic Workshop*, Portugal, 2024.
- [3] Gabriele Fiorentino, Giuseppe Quaranta, George Mylonakis, et al. Seismic Reassessment of the Leaning Tower of Pisa: Dynamic Monitoring, Site Response, and SSI. *35(2):703–736*, 2019.
- [4] COMSOL Multiphysics. Reference Manual, 2022.
- [5] Dionis Butuc, Anna De Falco, Filippo Landi, et al. Sensitivity analysis on the FE Model Parameters of the Leaning Tower of Pisa. In *Proc. of 20th International Probabilistic Workshop*, Portugal, 2024.
- [6] Edwin Reynders, Mattias Schevenels, and Guido De Roeck. *MACEC 3.4, the Matlab toolbox for experimental and operational modal analysis. Report BWM-2021-04*, 2021.
- [7] Bart Peeters and Guido De Roeck. Reference-based stochastic subspace identification for output-only modal analysis. *Mechanical Systems and Signal Processing*, 13(6):855–878, 1999.
- [8] Edwin Reynders, Jeroen Houbrechts, and Guido De Roeck. Fully automated (operational) modal analysis. *Mechanical Systems and Signal Processing*, 29:228–250, 2012.
- [9] Jhon P. Wolf. *Dynamic Soil-Structure interaction*. Prentice-Hall, New Jersey, 1985.
- [10] Jonathan P. Stewart, Raymond B. Seed, and Gregory L. Fenves. Seismic Soil-Structure Interaction in Buildings. II: Empirical Findings. *Journal of Geotechnical and Geoenvironmental Engineering*, 125(1):38–48, 1999.
- [11] D. Pellegrini, A. Barontini, M. Girardi, et al. Effects of temperature variations on the modal properties of masonry structures: An experimental-based numerical modelling approach. *Structures*, 53: 595–613, 2023.