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A design map for a structural health monitoring system based on redundancy of autocovariance functions

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ABSTRACT

Damage in a structure can be detected using vibration measurements under unknown random excitation. Structural health monitoring (SHM) systems must be often designed before knowledge about future damage. Also, the excitation characteristics are unknown and may vary between measurements. The objective of the present paper is to facilitate the design of an SHM system, when autocovariance functions (ACFs) are used as damage-sensitive features. Since the ACFs have the same form as a free decay of the system for a stationary random process, they are spatiotemporally correlated. Therefore, it is possible to achieve redundant measurement data. The data space can then be divided into two subspaces, a signal space and the noise space. The information lies in the signal space, while the noise space consists mostly of measurement errors. If the structure is damaged, the new data also hit the noise space, where damage can be detected. To this end, the objective is to pursue data redundancy. The degree of redundancy of the ACF data depends on three design parameters: (1) the number of active modes, (2) the number of sensors, and (3) the model order used in the data analysis. It is also shown that with a proper choice of the design parameters, the variability of the excitation statistics has no effect on redundancy. To help with the selection of the parameters, a design map is provided. It has four different regions, depending on the number of sensors and the model order: (1) both frequency and mode shape changes can be detected, (2) frequency changes can only be detected, (3) mode shape changes can only be detected, and (4) damage is not detected. A numerical experiment was conducted to verify the regions of the design map. The rank of the noiseless data matrix was used to indicate the degree of redundancy. With the help of the design map, the user can select the number of sensors and the model order to detect both frequency and mode shape changes.

Keywords: Damage detection, Autocovariance function, Spatiotemporal correlation, Design map, Number of sensors, Model order, Excitation statistics.

1. INTRODUCTION

Operational modal analysis (OMA) is based on covariance functions and an assumption of random excitation. The resulting modal parameters can be used as damage-sensitive features. Covariance functions can be also directly utilized in damage detection.

This paper focuses on autocovariance functions (ACFs) estimated for each sensor independently. If the process is stationary random, the ACFs have the same mathematical form as a free decay of the structure [1]. Therefore, ACFs consist of necessary dynamic characteristics of the structure to be used in damage detection. In wireless sensor networks (WSNs), ACFs are most appealing, because cross-covariance functions are not used. Each ACF can be estimated in the sensor node, and instead of transmitting long measurement records, only short ACFs need to be transmitted to the central node. In addition, ACFs do not need simultaneous sampling, which is a great advantage, because time synchronization in WSNs needs special attention.

Autocorrelation functions for damage detection have been suggested e.g. in [2–5]. The validity of ACFs in damage detection is still missing and no practical guidelines exist to select the design parameters of a structural health monitoring (SHM) system for a given structure.

The objective of this paper is to show that ACFs are valid features in damage detection. This is done by utilizing the spatiotemporal correlation property of ACFs, resulting in redundant data and making damage detection possible. Also, a tool to help the selection of important design parameters is provided based on data redundancy.

The paper is organized as follows. Autocovariance functions are introduced, and conditions of a redundant system are given. A design map is provided to select the important SHM system parameters. Data redundancy is then utilized in damage detection. A numerical experiment is performed to verify the theoretical findings. Finally, concluding remarks are given.

2. AUTOCOVARIANCE FUNCTIONS IN DAMAGE DETECTION

2.1. Autocovariance functions

The autocovariance function of a zero-mean ergodic random process $\{x(t)\}$ at lag τ is defined as

$$R_{xx}(\tau) = E[x(t)x(t+\tau)] \tag{1}$$

ACFs are often estimated using an FFT-based algorithm [6]. In a sensor network, ACFs are functions of space and time. Due to a special mathematical form of the ACFs, the measurement system can be designed redundant.

For a stationary random process with white noise excitation, the ACF of a displacement degree-of-freedom i is [7]

$$R_{ii}(\tau) = \sum_{r=1}^{n}\sum_{s=1}^{n} \phi_{ir}\phi_{is}c_{rs}[F_{rs}\cos(\omega_{dr}\tau) + G_{rs}\sin(\omega_{dr}\tau)]\exp\left(-\zeta_r\omega_r\tau\right) \tag{2}$$

where m_r, ω_r, ω_{dr}, ζ_r, and ϕ_{ir} are, respectively, the modal mass, the undamped natural circular frequency, the damped natural circular frequency, the damping ratio, and the ith component of the mode shape vector $\boldsymbol{\phi}_r$ of the mode r. Also,

$$\begin{aligned} F_{rs} &= \frac{1}{2m_r\omega_{dr}m_s\omega_{ds}}\left[\frac{-\zeta_r\omega_r - \zeta_s\omega_s}{\left(\zeta_r\omega_r + \zeta_s\omega_s\right)^2 + (\omega_{dr}+\omega_{ds})^2} + \frac{\zeta_r\omega_r + \zeta_s\omega_s}{\left(\zeta_r\omega_r + \zeta_s\omega_s\right)^2 + (\omega_{dr}-\omega_{ds})^2}\right] \\ G_{rs} &= \frac{1}{2m_r\omega_{dr}m_s\omega_{ds}}\left[\frac{\omega_{dr}+\omega_{ds}}{\left(\zeta_r\omega_r + \zeta_s\omega_s\right)^2 + (\omega_{dr}+\omega_{ds})^2} + \frac{\omega_{dr}-\omega_{ds}}{\left(\zeta_r\omega_r + \zeta_s\omega_s\right)^2 + (\omega_{dr}-\omega_{ds})^2}\right] \end{aligned} \tag{3}$$

and

$$c_{rs} = \boldsymbol{\phi}_r^T \mathbf{B}\mathbf{Q}\mathbf{B}^T \boldsymbol{\phi}_s \tag{4}$$

Note that all information about excitation is contained in the terms c_{rs} for mode pairs r and s. **B** is the load distribution matrix and **Q** is a diagonal matrix with mean square values of the independent loads. Both matrices **Q** and **B**, and consequently c_{rs}, can vary between measurements, because the number, spatial distributions, magnitudes, and frequency contents of the ambient loads may differ.

The ACF (2) can also be written as

$$R_{ii}(\tau) = \sum_{r=1}^{n} [A_{ir}\cos(\omega_{dr}\tau) + B_{ir}\sin(\omega_{dr}\tau)]\exp\left(-\zeta_r\omega_r\tau\right) \tag{5}$$

where

$$A_{ir} = \sum_{s=1}^{n} F_{rs}\phi_{ir}\phi_{is}\,c_{rs} \quad \text{and} \quad B_{ir} = \sum_{s=1}^{n} G_{rs}\phi_{ir}\phi_{is}\,c_{rs} \tag{6}$$

The ACF, evaluated at another lag $\tau_k = \tau + k\Delta t$, Δt being the time increment, is:

$$R_{ii}(\tau_k) = \sum_{r}\sum_{s} \phi_{ir}\phi_{is}c_{rs}\left[F_{rs}^{(k)}\cos(\omega_{dr}\tau) + G_{rs}^{(k)}\sin(\omega_{dr}\tau)\right]\exp\left(-\zeta_r\omega_r\tau\right) \tag{7}$$

where $F_{rs}^{(k)}$ and $G_{rs}^{(k)}$ are constants:

$$\begin{aligned} F_{rs}^{(k)} &= [F_{rs}\cos(\omega_{ds}k\Delta t) + G_{rs}\sin(\omega_{ds}k\Delta t)]\exp\left(-\zeta_s\omega_s k\Delta t\right) \\ G_{rs}^{(k)} &= [-F_{rs}\sin(\omega_{ds}k\Delta t) + G_{rs}\cos(\omega_{ds}k\Delta t)]\exp\left(-\zeta_s\omega_s k\Delta t\right) \end{aligned} \tag{8}$$

Note that Equation (7) has the same mathematical form as (2), which can be used for temporal redundancy.

For velocities or accelerations, the ACFs have similar expressions. An example of an autocovariance function with six active modes is shown in Figure 1.

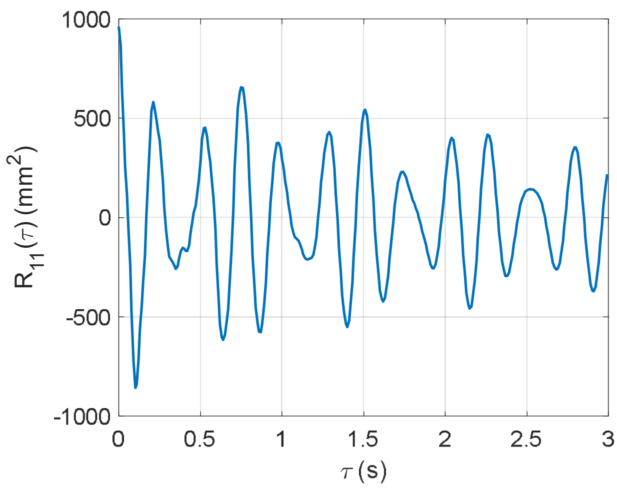


Figure 1. An autocovariance function with 300 lags. The number of modes is 6.

2.2. Redundancy

Data redundancy is important in damage detection, discussed in Section 2.3. The three parameters that determine the degree of redundancy are:

- n = the number of active modes
- p = the number of sensors
- m = the model order, or the number of time-shifted ACFs in the analysis.

The data equations in the cases that follow, can be written in a matrix form:

$$\{\mathbf{R}(\tau_k)\} = [\mathbf{A}]\{\mathbf{q}(\tau)\}, \qquad k = 0, \dots, m \tag{9}$$

where $\{\mathbf{R}(\tau_k)\}$ is the ACF data vector with a length of $p(m+1)$, $\{\mathbf{q}(\tau)\}$ are the independent time functions, and $[\mathbf{A}]$ is a coefficient matrix, which should be tall and time-independent for redundancy.

2.2.1. Spatial redundancy

Spatial correlation refers to ACFs from different sensors at the same lag τ ($m = 0$). In Equation (2), the basis vectors are the element-wise products of the mode shape vectors $\boldsymbol{\phi}_r \odot \boldsymbol{\phi}_s$. For n modes, the number of different combinations is $n(n+1)/2$, resulting in the redundancy condition

$$p > \frac{1}{2}n(n+1) \tag{10}$$

2.2.2. Temporal redundancy

Temporal redundancy refers to correlation of an ACF at different lags $\tau_k, k = 0, \dots, m$. Correlation between sensors is omitted. Using Equation (5) and the similar mathematical form of the shifted ACFs (7), it can be shown that for each mode, two lags are needed to compute the value of the ACF at a third lag. In total, $2n + 1$ data points are needed to make a single ACF redundant. The condition for redundancy thus reads

$$m \geq 2n \tag{11}$$

2.2.3. Spatiotemporal redundancy

In spatiotemporal correlation, both the space and time dimensions are used to make the data redundant. Equation (2), which is a double sum, has $2n^2$ terms (coefficient 2 is due to a sine and cosine function in each term). The condition for redundancy is: $p(m+1) > 2n^2$, from which

$$m > \frac{2n^2}{p} - 1 \tag{12}$$

2.3. Design map

The redundancy conditions can be transformed into a visual form for better usability. The result is a design map with four regions depending on the design parameters n, p, and m (Figure 2):

- Cyan: spatial redundancy. Mode shape changes can be only detected.
- Blue: temporal redundancy. Frequency changes can be only detected.
- Green: spatiotemporal redundancy. Both frequency and mode shape changes can be detected.
- Red: non-redundant data. Detection is questionable.

The two marked coordinates in the map are

$$p_1 = \frac{1}{6}n(3n+1) \quad \text{and} \quad p_2 = \frac{1}{2}n(n+1) \tag{13}$$

where p_1 is a special point, for which spatiotemporal redundancy is achieved with a smaller model order ($m = 2$) than given by Equation (12).

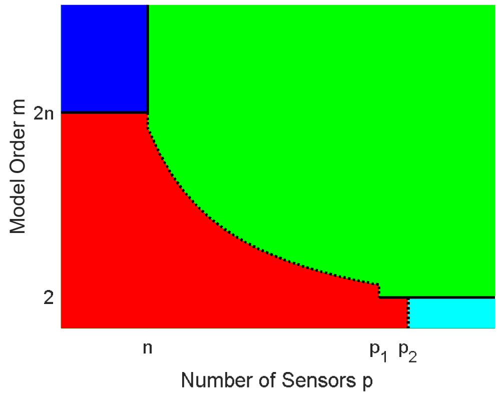


Figure 2. Design map.

2.4. Damage detection utilizing redundancy

Data redundancy means that high-dimensional data span only a smaller-dimensional subspace. Denote this subspace as the signal space. The remaining subspace is denoted as the noise space, which for noiseless data is empty. It is assumed that if damage occurs, the new data will hit also the noise space of the training data. Damage can be detected more probably in the noise space having small variance than in the signal space with a large variance.

The steps in the damage detection algorithm are briefly described. More details can be found in [8]. ACFs are estimated from each sensor and measurement individually. The training data (ACFs from the healthy structure) are subjected to whitening. All data are transformed with the same whitening matrix. The data are projected onto the first principal component to find the most statistically significant change. Generalized extreme value statistics (GENV) distributions [9] of the first principal component scores of the training data are identified. Extreme value statistics (EVS) control charts are plotted with the allowed probability of false positives determining the control limits [10].

3. NUMERICAL EXPERIMENT

A finite element model of a bridge deck with a concrete slab and steel girders was built to generate ACFs from 28 accelerometers and study detection of an open crack in a steel girder (Figure 3). The ACFs were synthesized using the modal parameters obtained from the eigenvalue analysis and Equation (2). The number of modes was n = 6. The data were noiseless so that the data rank could be evaluated to verify the redundancy conditions. Excitation statistics between measurements varied due to random parameters c_{rs} (Equation (4)).

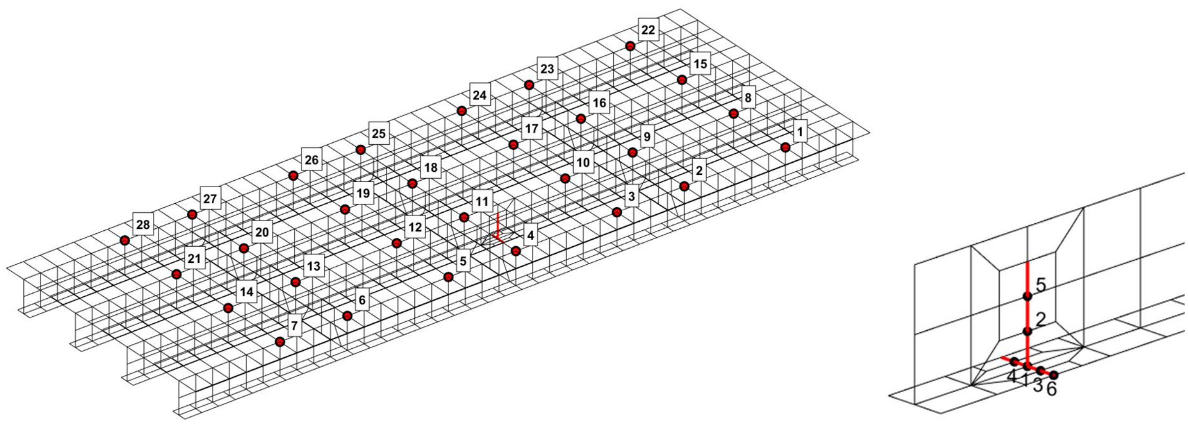


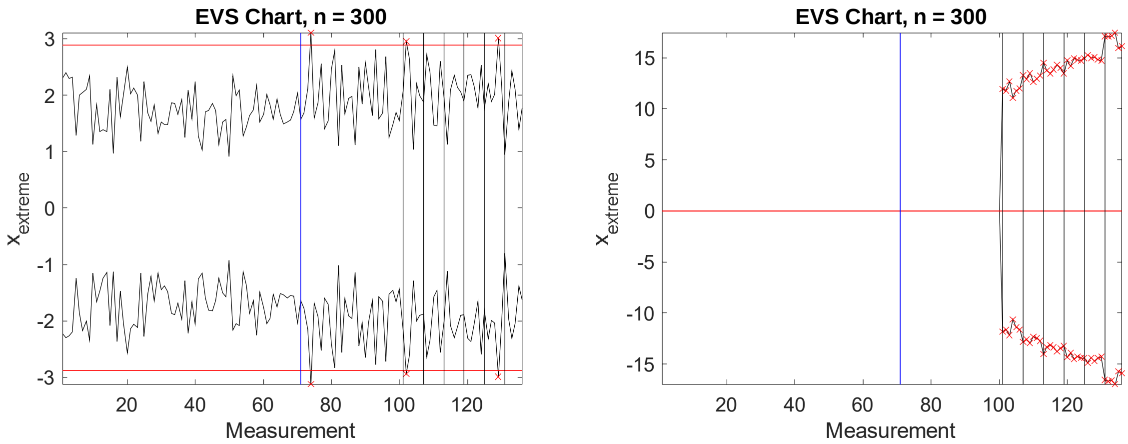
Figure 3. A finite element model of the bridge deck with 28 accelerometers. An open crack with a gradually increasing length in a girder between sensors 10 and 11.

To study the effects of different redundancies on damage detection, the changes due to damage were divided into (1) frequency changes only, keeping mode shapes unchanged, and (2) mode shape changes only, keeping frequencies unchanged. Different cases were designed using the design map (Figure 2).

The first 100 data sets were from the healthy structure, while the next 36 data sets were from the damaged structure. Each six crack lengths were monitored with six measurements. The training data included the first 70 data sets. Extreme values of subgroups were stored, each subgroup including data from a single measurement. The training data and different damage levels are indicated in the control charts. Logarithmic scaling was applied for clarity.

3.1. Spatial redundancy

First, spatial redundancy was studied with p = 22 and m = 0. The data dimensionality was 22 and the rank was 21, implying redundant data. EVS control charts in Figure 4 show that frequency changes remained undetected, whereas mode shape changes were perfectly detected, agreeing with the theory.

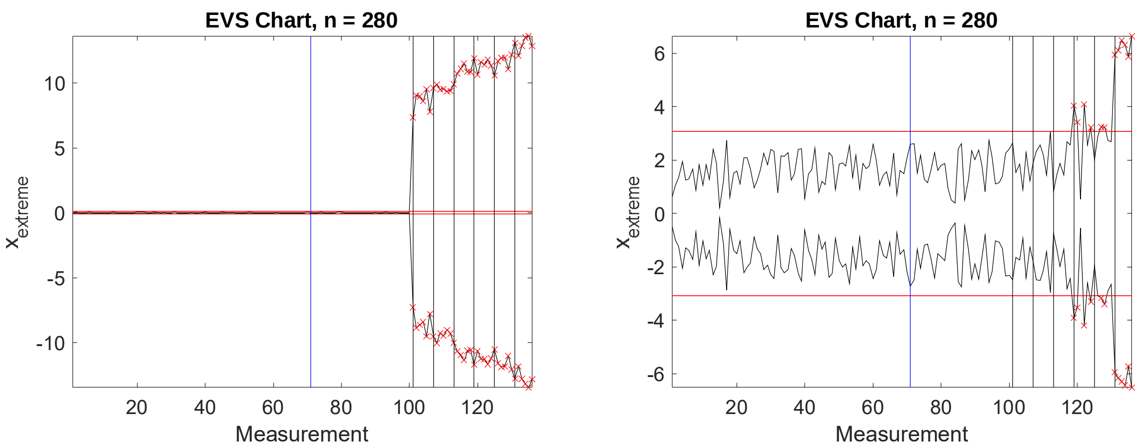


(a) (b)

Figure 4. EVS control charts using spatial redundancy. a) Frequency changes, b) mode shape changes.

3.2. Temporal redundancy

In the second case, temporal redundancy was studied with p = 4 and m = 20. The data dimensionality was 84 and the rank was 48, implying redundancy. Control charts in Figure 5 show that frequency changes were perfectly detected, while the most mode shape changes remained undetected, which agreed with the theory.

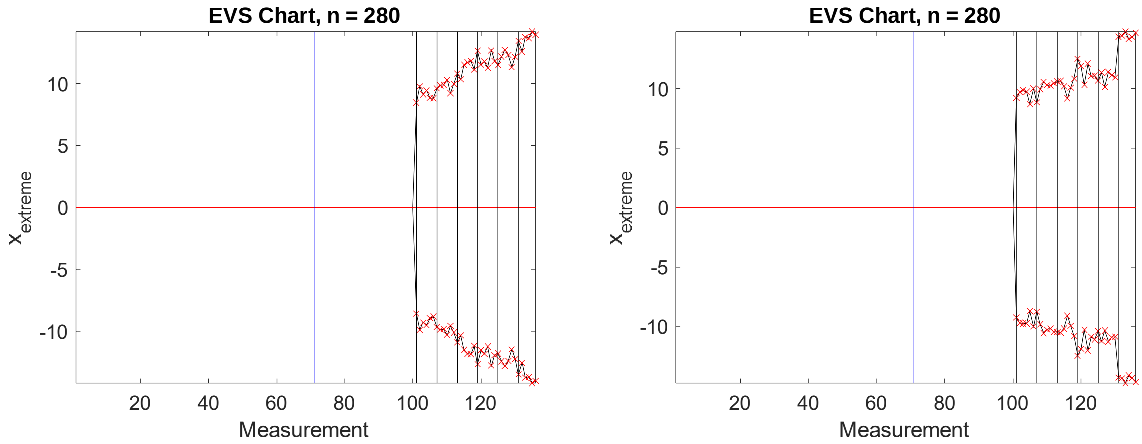


(a) (b)

Figure 5. EVS control charts using temporal redundancy. a) Frequency changes, b) mode shape changes.

3.3. Spatiotemporal redundancy

Third, spatiotemporal redundancy was studied with $p = 8$ and $m = 20$. The data dimensionality was 168 and the rank was 72, implying redundant data. EVS control charts in Figure 6 show that both frequency and mode shape changes were perfectly detected, which agreed with the theory.

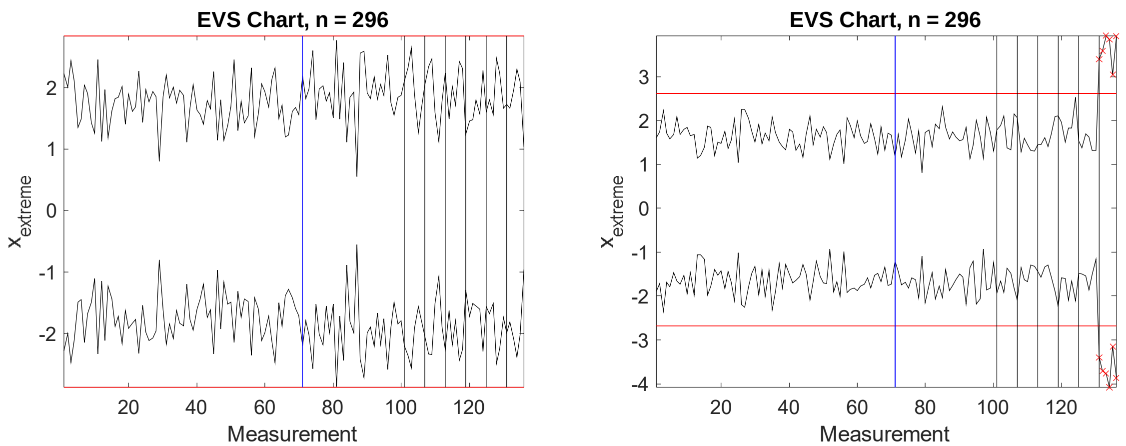


(a) (b)

Figure 6. EVS control charts using spatiotemporal redundancy. a) Frequency changes, b) mode shape changes.

3.4. Non-redundant data

Non-redundant data were analyzed in the fourth case with $p = 4$ and $m = 4$. Both the data dimensionality and the rank were equal to 20. Figure 7 shows that frequency changes could not be detected, and the largest crack was only detected from the mode shape changes.



(a) (b)

Figure 7. EVS control charts using non-redundant data. a) Frequency changes, b) mode shape changes.

4. CONCLUSIONS

Autocovariance functions were shown to be valid damage-sensitive features in structural health monitoring. They contain the necessary dynamic characteristics of the structure for damage detection. They are spatiotemporally correlated, which can be utilized in the pursuit of redundant data. A design map was provided to facilitate the selection of the design variables. Spatiotemporal redundancy is recommended, because both frequency and mode shape changes can be detected with a manageable number of sensors. The process is assumed to be stationary random, but the variability of the excitation statistics has no effect on redundancy.

The ACFs studied in this paper were noiseless, but in reality, they must be estimated from long data records. In addition, no environmental influences were considered. It is expected that in a more realistic case, the number of sensors or the model order must be increased to take the environmental variability into account.

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