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Model Updating of Rotating Wind Turbines Using Operational Modal Analysis and Floquet Mode Decomposition

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ABSTRACT
The structural complexity of modern wind turbines, combined with numerous uncertain or unknown parameters, presents significant challenges for accurate predictive modeling. Model updating, which refines numerical model parameters using measurement data, offers a means to mitigate these discrepancies. While extensively applied to stationary structures, its extension to rotating wind turbines remains limited, as their time-periodic dynamics violate key assumptions underlying conventional methods.
This study develops a numerical framework for model updating of rotating wind turbines based on an equivalent Linear Time-Invariant (LTI) approximation, derived through a Fourier decomposition of the system's Floquet modes. A simplified 5 Degrees of Freedom (DoF) turbine model is employed to evaluate the effectiveness of a deterministic model updating strategy leveraging this approximation. Synthetic vibration data, generated from the model using a predefined parameter set, serve as reference measurements for assessing parameter recovery accuracy. Modal features extracted via Operational Modal Analysis (OMA) are used to construct the cost function that quantifies discrepancies between predicted and observed modes.
The results underscore the potential of equivalent LTI representations in facilitating model updating for rotating systems, as they effectively capture the modal characteristics identified via OMA. This study establishes a foundation for extending this methodology to more complex, industrial-scale wind turbine models, provided that the computational cost of model evaluation remains manageable.

Keywords: OMA, Model updating, Wind turbine, Vibration-based, Floquet modes, LTP system

1. INTRODUCTION

The transition to renewable energy has driven the rapid development of wind turbines with increased capacities, longer blades, and taller towers. While these innovations have significantly advanced wind power as a key contributor to sustainable energy production, they have also introduced considerable challenges in structural modeling. The inherent complexity of wind turbines, combined with their multi-physics nature, limits the accuracy of predictive models. In particular in the context of monitoring with digital twins, where models representing the operating structure may have many uncertain or even unknown parameters, such as material properties, geometric configurations, mass, and stiffness.

Model updating, which involves refining model parameters based on real-structure measurement data [1], provides a promising approach to address these challenges. In structural dynamics, this methodology has been extensively applied to stationary structures such as bridges and buildings, typically using Operational Modal Analysis (OMA) to extract modal characteristics from in situ vibration measurements, followed by modal-based model updating. However, wind turbines present unique difficulties due to their time-periodic behavior induced by rotor rotation [2]. Unlike stationary systems, the mode shapes of wind turbines become inherently time-periodic, which can be effectively described through the Floquet formalism [3].

Although there is growing interest in vibration-based model updating methodologies for wind turbines, most studies have focused on non-operating turbines [4] or non-rotating components such as towers and support structures [5–8], avoiding the complexities introduced by time-periodic dynamics. While the time-periodic nature of operational wind turbines is well recognized, its explicit consideration in model updating remains underexplored. Recent research has demonstrated that these systems can be approximated by equivalent Linear Time-Invariant (LTI) models using Floquet and Fourier decompositions [9]. This enables the application of classical model updating techniques, thereby addressing the challenges posed by their dynamic behavior.

This study develops a numerical framework for model updating of rotating wind turbines, leveraging equivalent LTI systems to address their time-periodic dynamics. The framework is evaluated on a simplified 5 Degree of Freedom (DoF) academic turbine model to assess its robustness and effectiveness. Through this work, we provide new insights into model calibration for rotating structures, emphasizing the necessity of accounting for time-periodic dynamics in model updating techniques.

2. MODEL UPDATING OF A ROTATING WIND TURBINE USING AN LTI APPROXIMATION

In this study, synthetic vibration data (i.e. generated by a numerical model) will be considered as reference. To achieve an accurate model update, it is essential to define relevant features that quantitatively assess the model's ability to replicate the observed data. In the context of vibration analysis, these features typically consist of modal frequencies and mode shapes. OMA is frequently employed to extract these features from measured vibration data, pooled in the vector $\bar{d}$, under the assumption that the system behaves according to an LTI framework.

2.1. LTI approximation of an LTP system using a Floquet-Fourier decomposition

In the operating state of the turbine, the blades rotate with angular velocity Ω, making the system time periodic. While Ω is assumed to be constant in this study, real operating conditions involve slight variations, which are considered negligible. This time-periodic behavior gives rise to Floquet modes, which describe the dynamic response of systems with periodic coefficients. The precise formulation of such systems requires the use of Floquet theory [10, 11].

Specifically, the autonomous part of an LTP system is described by the following differential equation:

$$\dot{x}(t) = A(t)x(t) \tag{1}$$

where $x \in \mathbb{R}^n$ represents the state vector of the system, and $A \in \mathbb{C}^{n\times n}$ is a matrix that is periodic with period $T = \frac{2\pi}{\Omega}$. The Lyapunov-Floquet transformation allows for the computation of the eigenvalues and eigenvectors $v_k(t)$ of the system. Each eigenvector, $v_k(t)$, is a time-dependent vector of dimension n and is periodic with period T. Therefore, it can be expanded as a Fourier series:

$$v_k(t) = \sum_{j=-\infty}^{\infty} V_{k,j} e^{i(j-j_k)\Omega t} \tag{2}$$

where $V_{k,j} \in \mathbb{C}^n$ contains the j^{th} Fourier coefficients corresponding to each state variable, and j_k is a constant coefficient associated with the Lyapunov-Floquet transformation (see [11] for further details).

Although the Fourier series decomposition of the Floquet modes theoretically results in an infinite number of Floquet-Fourier harmonics indexed by j, only a finite number of these harmonics typically exhibit significant amplitudes. As shown e.g. in [9], an LTP system can be approximated by an LTI system constructed from a truncated series of Fourier components, $V_{k,j}$, corresponding to the Floquet modes $v_k(t)$. Furthermore, these modes can be identified using standard OMA algorithms, such as the Stochastic Subspace Identification (SSI) algorithm [9].

The participation factor allows to estimate the contribution of each harmonic to the overall response. For the j^{th} harmonic of the k^{th} Floquet mode, it is defined as $\Phi_{k,j} = \|V_{k,j}\| / \sum_{i=-\infty}^{\infty} \|V_{k,i}\|$, where $\|\cdot\|$ is the norm associated with the dot (hermitian) product $\langle a, b\rangle = a^H b$.

The truncated Fourier series will be constructed by retaining the Fourier-Floquet harmonics $V_{k,j}$ with participation factors greater than 1 %. These harmonics are time-independent mode shapes, each associated with a specific frequency $f_{k,j}$. Collectively, they form a modal basis $\{V_m, f_m\}$ of an equivalent LTI system of dimension $m > n$, where m is the number of harmonics retained in the truncation of the infinite series.

2.2. Determinist optimization of the model properties

The next step involves selecting the set of model parameters θ_M that will be updated. Choosing an appropriate set for θ_M is a challenging task; it must adequately represent the unknown structural properties while remaining sufficiently constrained to avoid ill-conditioning issues [1].

The deterministic approach aims to identify an optimal set of model parameters θ_M^* that minimizes the discrepancy between the experimental data $\bar{d}$ and the model predictions $G_M(\theta_M)$. This misfit is quantified by a cost function F, which transforms the model updating problem into the following optimization problem:

$$\theta_M^* = \underset{\theta_M \in D_M}{\arg\min}\, F(G_M(\theta_M), \bar{d}) \tag{3}$$

with D_M the model parameters space. In this study, the cost function is defined as a weighted least squares fit between the modes shapes and modal frequency of both the model predictions and the experimental data:

$$F(\theta_M) = \frac{1}{2}\sum_{r=0}^{p} \frac{\left(\overline{f}_r - f_r(\theta_M)\right)^2}{\left(3\sigma(\overline{f}_r)\right)^2} + \frac{1}{2}\sum_{r=0}^{p} \frac{\left\|\gamma_r \overline{V}_r - V_r(\theta_M)\right\|^2}{\left\|3\gamma_r\sigma(\overline{V}_r)\right\|^2}, \tag{4}$$

where $\{V_r, f_r\}$ are the modal properties predicted by the LTI model. $\{\overline{V_r}, \overline{f_r}\}$ are the modal properties extracted from the data $\bar{d}$ using SSI, while $\sigma(\cdot)$ denotes the standard deviation associated with these properties, also estimated via SSI. γ_r is the normalized coefficient, which ensures that the mode shapes are comparable, with $\gamma_r = \overline{V}_r^\top V_r(\theta_M)/\|\overline{V}_r\|_2^2$. To prioritize more reliable modes, we choose to weight the modes in the cost function by their corresponding standard deviation.

3. APPLICATION TO A 5 DOF WIND TURBINE MODEL

3.1. Model presentation and LTI approximation verification

A simplified wind turbine model with 5 DoF, depicted in Figure 1, is used to apply the proposed framework. A comprehensive description of this model can be found in [11]. The turbine operates with a constant rotational speed of $\Omega = 1.4$, rad/s. The parameters to be updated are selected as the rotational stiffness of each blade, denoted as $\theta_M = \{G_1, G_2, G_3\}$.

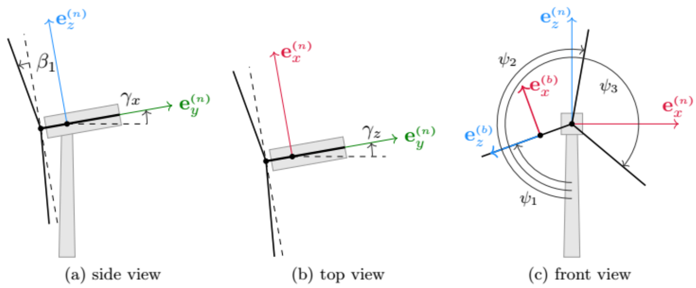


Figure 1: Simplified wind turbine model with 5 DoF.

An arbitrary set of these parameters, $\theta_d = \{8.6, 7.3, 9\} \times 10^7\,\mathrm{Nm}$, is defined to simulate a hypothetical "real" turbine. Using this fixed parameter set, the model generates noisy acceleration time series for each DoF, simulating the vibration measurements that could be obtained from an operational turbine. Then, the SSI method is applied on these time series, leading to the stabilization diagram on Figure 2. This identification process provides the modal properties $\overline{d} = \{\overline{V}, \overline{f}\}$ of this "real" turbine, along with their associated standard deviation, listed in Table 1.

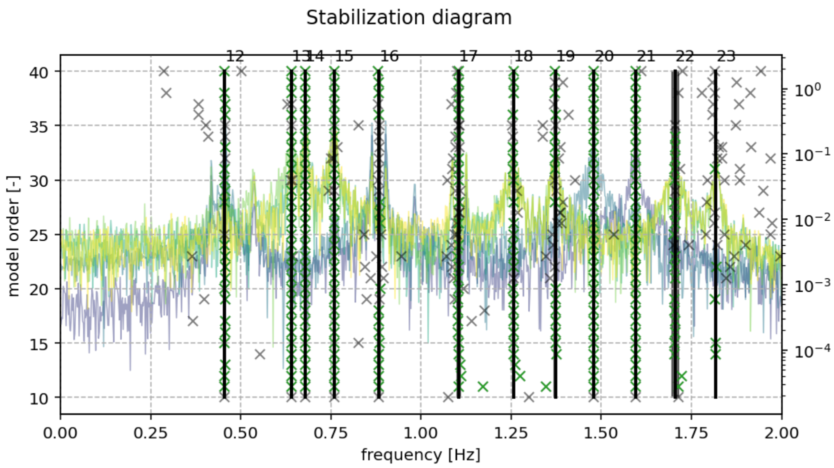


Figure 2: Stabilization diagram from acceleration measures of all of the 5 DoF.

The identified frequencies are compared to those of the LTI equivalent system derived using the process described in Section 2.1.. This comparison demonstrates the validity of the approximation, as the identified frequencies closely match the estimated ones. Figure 3 illustrates the 12 mode shapes identified through SSI and those computed from the LTI approximation. This comparison confirms that the approximation is successful not only in predicting the modal frequencies but also in accurately capturing the mode shapes, as identified by OMA on the LTP system.

However, a slight discrepancy is observed in the frequency of mode 5, highlighting that the LTI system serves only as an approximation of the real system. Similarly, in Figure 3, the approximated mode shapes 5 and 6 exhibit some divergence from those identified via OMA. This indicates that the dynamics of the turbine in these modes are more complex and less accurately captured by the LTI approximation.

Table 1: Modal frequencies of the turbine identified through SSI, along with their associated variability. These are compared to the frequencies of the approximated LTI system obtained via Fourier-Floquet decomposition.

Mode	LTI-approximated system frequencies [Hz]	OMA on time series frequencies$\pm$3 std [Hz]
1	0.457	0.456 $\pm$ 0.002
2	0.641	0.641 $\pm$ 0.001
3	0.680	0.680 $\pm$ 0.001
4	0.761	0.760 $\pm$ 0.001
5	0.864	0.884 $\pm$ 0.005
6	1.086	1.104 $\pm$ 0.006
7	1.257	1.257 $\pm$ 0.004
8	1.376	1.372 $\pm$ 0.004
9	1.480	1.479 $\pm$ 0.001
10	1.599	1.596 $\pm$ 0.001
11	1.700	1.700 $\pm$ 0.010
12	1.822	1.817 $\pm$ 0.005

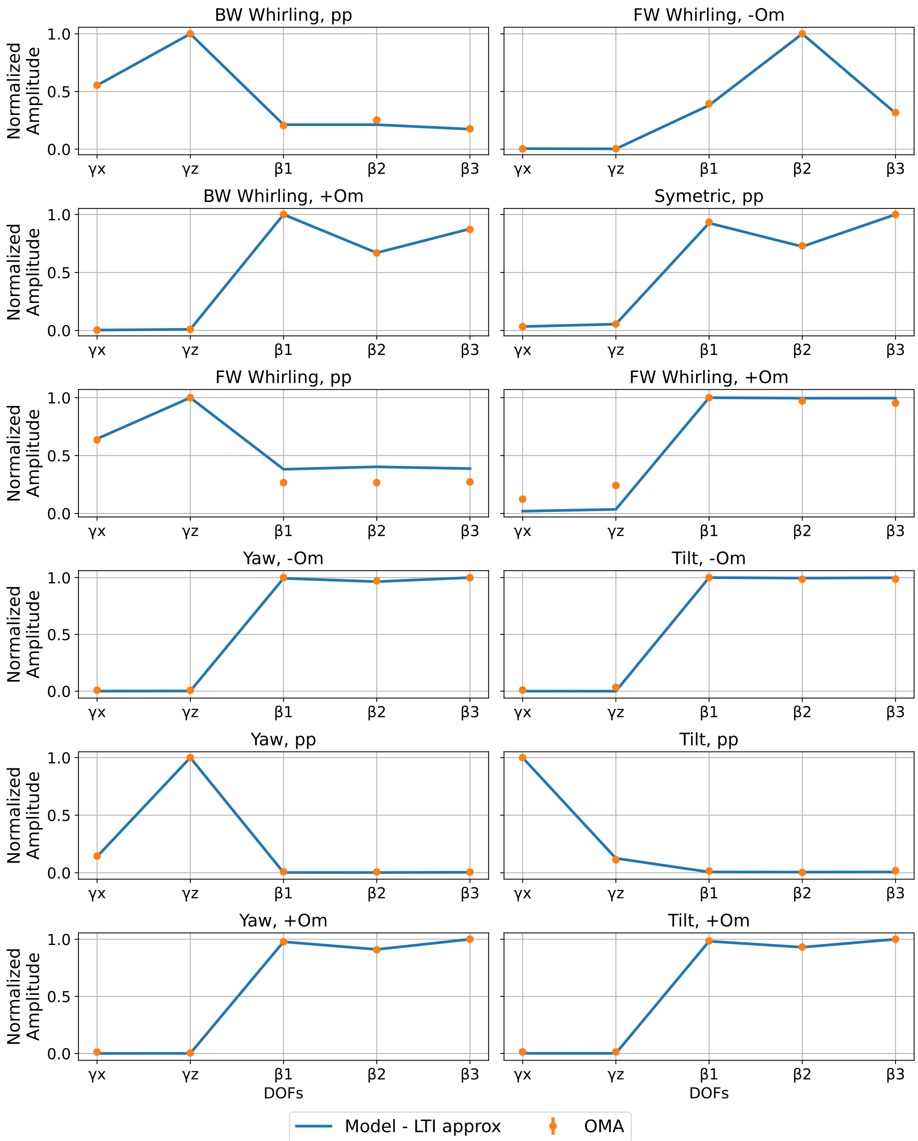


Figure 3: Mode shapes identified through OMA compared to the mode shapes of the LTI-approximated system.

3.2. Model Updating

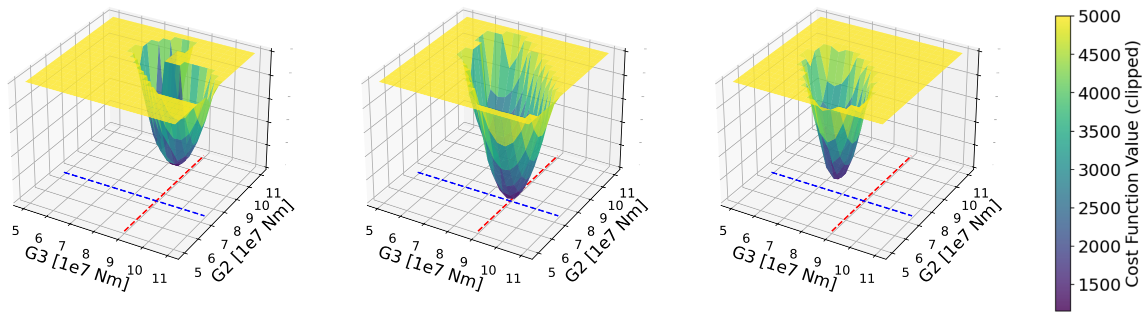
In this section, the set of parameters θ_d is treated as unknown. Our objective is to estimate these parameters by employing the methodology outlined in Section 2.2. The 5 DoF model is used to predict the modal properties of the approximated LTI system as a function of θ_M. The dimension m of this new modal basis depends on the model parameters θ_M. In fact, the loss of rotor isotropy leads to the emergence of more remaining harmonics in the truncation. This reflects the fact that an anisotropic rotating rotor is a more complex system than an isotropic one. However, for the cost function (4) to be valid, $p_{\overline{d}}$ and p_M must be the same. Here, p is chosen as $p = 12$, which correspond to the number of modes identified via SSI. Furthermore, to properly compare the modes, they must be ordered identically. Therefore, a sorting algorithm is employed to select and order the modes, using the MAC/frequency distance between a predicted mode k and an experimental mode l, defined as:

$$MAC/freq(\phi_k(\theta_M), \overline{\phi}_l) = 1 - MAC(V_k(\theta_M), \overline{V}_l) + \frac{f_k(\theta_M) - \overline{f}_l}{f_k(\theta_M) + \overline{f}_l} \tag{5}$$

where MAC refers to the Modal Assurance Criteria, as defined in [12].

Table 2: Modal frequencies of the "real" turbine identified through SSI, along with their associated variability, compared to the predicted frequencies before and after the model updating process.

Mode	Frequencies		
	"Real" turbine ±3.std [Hz]	Predicted before updating [Hz]	Predicted after updating [Hz]
1	0.456 ± 0.002	0.448	0.457
2	0.641 ± 0.001	0.643	0.641
3	0.680 ± 0.000	0.670	0.680
4	0.760 ± 0.000	0.744	0.760
5	0.884 ± 0.005	0.866	0.863
6	1.104 ± 0.006	1.086	1.086
7	1.257± 0.004	1.247	1.256
8	1.372 ± 0.004	1.369	1.375
9	1.479 ± 0.001	1.470	1.479
10	1.596 ± 0.001	1.592	1.598
11	1.70 ± 0.01	1.69	1.70
12	1.817 ± 0.005	1.815	1.821



$G_1 = 7.2 \times 10^7$Nm $\quad$ $G_1 = 8.5 \times 10^7$Nm $\quad$ $G_1 = 10 \times 10^7$Nm

Figure 4: Evolution of the cost function (4), clipped at $F = 5000$, with the parameters value $\theta_M = \{G_1, G_2, G_3\}$, compared to the "real" value of G_2 in blue and G_3 in red.

The shape of the cost function (4) for this model, with the experimental data $\overline{d}$ identified through the SSI, is shown in Figure 4. The cost function is observed to be sharply concentrated around the real value of the parameters, indicating that it is well-defined. Furthermore, no local minima are present, which suggests that the optimization process—and thus the model updating—will be efficient and quick.

The next step is to solve the optimization problem (3), starting with an initial set of parameters chosen as $\theta_{Mini} = \{8, 8, 8\} \times 10^7\text{Nm}$, which represents a case of an isotropic rotor. Each evaluation of the model $G(\theta_M)$ can be relatively costly due to the need to compute the state transition matrix of the system over one full period, required for applying the Lyapunov-Floquet transformation. To address this, an optimization algorithm that limits the number of model evaluations is used: COBYQA, which is a derivative-free trust-region Sequential Quadratic Programming (SQP) method based on quadratic models obtained through underdetermined interpolation [13].

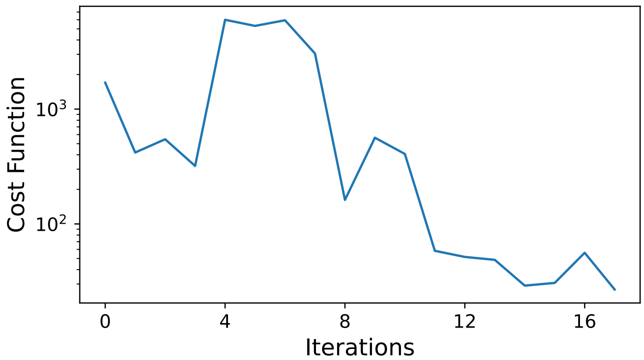


Figure 5: Evolution of the cost function (4) with the number of iterations of the optimization algorithm COBYQA.

Figure 5 illustrates the evolution of the cost function (4) at each call to the function F required for the optimization. The optimization converges effectively in terms of computation cost, thanks to the limited number of model evaluations. As shown in Table 3, the cost function does not reach zero, attesting to the residual discrepancy in Table 2 between the updated modes and the identified ones.

To assess the effectiveness of this updating method, we can verify that the updated parameters align with the ”real” ones. The mean relative error for the parameters is defined as: $\bar{\epsilon} = \frac{1}{N}\sum_{i=0}^{N} \frac{\theta_d(i) - \theta_M(i)}{\theta_d(i)}$ and the maximal absolute error as: $\boldsymbol{\epsilon} = \max_{i \in \{0,N\}}(\theta_d(i) - \theta_M(i))$, where θ_M represents the parameters to be updated, and θ_d represents the parameters used to generate the synthetic dataset, N is the dimension of the parameters vector, here $N = 3$. Table 3 summarizes the effectiveness of the updating process.

Table 3: Assessment of the model updating technique.

Method	Nb of evaluation	$F(\theta_{Mini})$	$F(\theta_{Mup})$	$\bar{\epsilon}_{ini}$ %	$\boldsymbol{\epsilon}_{ini}$ [Nm]	$\bar{\epsilon}_{up}$ %	$\boldsymbol{\epsilon}_{up}$ [Nm]	Computational Time [s]
COBYQA	18	17×10^3	27	27.6	1×10^7	0.8	6×10^5	7.5

The mean relative error on the parameters is only 0.8%, which is satisfactory, particularly when compared to the uncertainties in the experimental data, which can reach up to 0.5% for certain frequencies. This result is even more notable considering that an LTI approximation was employed to update the model.

4. CONCLUSIONS

This work develops a new strategy for updating a rotating wind turbine model based on modal properties. Specifically, we utilized an LTI approximation of the system, incorporating a Floquet-Fourier decomposition to address the complexities induced by the system's periodic behavior. This simplification allows for the application of a classical deterministic updating process. The investigation, carried out on a simplified 5 DoF model, evaluated the validity of the LTI approximation in representing modes identified through OMA. By employing the equivalent LTI system, we successfully updated key model parameters, including blade rotational stiffness, within a reasonable computational time. Furthermore, the remaining discrepancies in the updated parameters were found to be minimal, considering the inherent approximations involved.

Despite the success of the deterministic approach, certain limitations were noted, particularly its inability to account for uncertainties in model predictions. To address this shortcoming, a probabilistic framework, such as the Bayesian method, should be incorporated to quantify uncertainties and provide a confidence interval for the updated parameters. While the current methodology proves effective for a simplified model, future work will focus on extending this approach to more complex and realistic systems. Moreover, when scaling up to industrial cases, an additional challenge arises regarding the computation time of Floquet modes, which must be reduced—potentially through the use of surrogate models.

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