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Addressing Mode-Mixing Challenges in Structural Health Monitoring: Numerical and experimental validation

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ABSTRACT

Multisensory data is essential for conducting comprehensive and accurate modal analysis in modern structural identification. However, this data is often contaminated with significant noise, which makes effective denoising crucial for reliable analysis. While traditional Multivariate Empirical Mode Decomposition (MEMD) is a powerful tool, it can introduce mode-mixing due to its sifting operations. This mode-mixing leads to inaccuracies in modal identification and decreases the reliability of the assessment of structural conditions. Addressing this limitation is vital for fully utilizing MEMD in structural health monitoring (SHM) and ensuring precise and high-fidelity analysis outcomes. To structure noise separation properly, singular spectrum analysis (SSA) is integrated with the MEMD to alleviate mode-mixing in the resulting modal responses. Unlike other techniques, SSA is data-adaptive and non-parametric, which means it does not require predefined assumptions about the frequency content of the signal. Furthermore, this decomposition process of SSA is based on SVD, which provides a robust foundation for extracting dominant modal components and filtering out noise with high precision. The effectiveness of the integrated MEMD-SSA technique is verified through comprehensive validation studies, including numerical simulations and a full-scale application using the Lysefjord Bridge dataset. This range of studies demonstrates that the technique performs reliably under various real-world conditions, addressing key challenges in SHM. The approaches employed in this study include detecting low-energy frequencies, mitigating mode-mixing from closely spaced modes, and managing significant measurement noise that can obscure modal characteristics. This noise-robust technique focuses on the accuracy and reliability of SHM, leading to improved safety and maintenance. Application to the Lysefjord Bridge dataset demonstrates the ability of this method to manage complex modal behaviour in a full-scale structure which is subjected to various external loadings.

Keywords: Multivariate, EMD, Modal Analysis, Structural Health Monitoring, SSA, Bridge dataset

1. INTRODUCTION

The origins of structural health monitoring (SHM) date back to the first instance of using dynamic response measurements to assess structural integrity, as demonstrated by Doebling et al. [1] in their seminal review on vibration-based damage identification techniques. SHM is a process that involves the continuous or periodic assessment of the condition of a structure using sensor data, analytical models, and diagnostic algorithms to detect damage, ensure safety, and optimize maintenance [2, 3]. A key aspect of SHM is modal analysis, which enables the characterization of dynamic properties such as natural frequencies, mode shapes, and damping ratios—quantities that provide critical insights into structural integrity and the presence of potential damage in bridges, buildings, and other infrastructure[4]. However, in real-world applications, modal analysis is often confounded by environmental and operational noise, sensor constraints, and mode-mixing phenomena, all of which complicate the extraction of meaningful dynamic features. Conventional signal processing techniques remain central to modal identification but are inherently susceptible to noise, which can obscure the underlying structural response. To address these limitations, an approach that integrates Multivariate Empirical Mode Decomposition (MEMD) [5] with Singular Spectrum Analysis (SSA) [6] has been employed. MEMD, an extension of Empirical Mode Decomposition (EMD), is particularly well-suited to multisensory data analysis, as it allows for the simultaneous processing of multiple signals. However, despite its advantages, MEMD is subject to mode-mixing, wherein intrinsic mode functions (IMFs) exhibit frequency components from multiple bands due to excessive sifting iterations, thereby affecting the reliability of modal parameter estimation.

In response to the limitations associated with MEMD, this study incorporates SSA as a complementary strategy for noise separation and mode enhancement. SSA, a data-driven decomposition technique, extracts underlying trends from time-series data without requiring prior assumptions about frequency content. This characteristic makes SSA particularly well-suited for isolating dominant modal components while suppressing noise. To evaluate the efficacy of the MEMD-SSA framework, a systematic validation process is undertaken, encompassing both numerical simulations and a real-world application using the Lysefjord Bridge dataset [7]. Characterized as a long-span suspension bridge, Lysefjord serves as an exemplary case study due to its *low-frequency modal characteristics* and *susceptibility to environmental noise*. Through this investigation, MEMD-SSA is demonstrated to be effective in detecting low-energy modes, minimizing interference from closely spaced modes, and improving the accuracy of modal parameter estimation under realistic operational conditions.

2. BACKGROUND THEORY

2.1. Multivariate EMD

Multivariate empirical mode decomposition (MEMD) extends the empirical mode decomposition (EMD) framework to analyze multidimensional signals through adaptive, data-driven decomposition. The fundamental concept of EMD is that any complex dataset can be decomposed into a finite number of intrinsic mode functions (IMFs). For a real values p–dimensional signal $x(t)$, the application of EMD yields M set of IMFs denoted as $\{d_i(t)\}_{i=1}^{M}$, and a monotonic residue $r(t)$. Eq.1 represents the decomposition of signal $x(t)$ into its respective IMFs.

$$x(t) = \sum_{i=1}^{M} d_i(t) + r(t) \tag{1}$$

While EMD proves effective for single-channel signal analysis, its multivariate extension encounters critical limitations when processing synchronized multichannel measurements. This approach neglects the joint information derived from multichannel measurements. The independent decomposition of individual channels introduces two fundamental issues: (1) **IMF count mismatch:** $M^{(k)} \neq M^{(l)}$ *for channels* $k \neq l$ and (2) **Cross-channel spectral misalignment:** $f_i^{(k)} \nsim f_j^{(l)}$, where $M^{(k)}$ denotes the IMF quantity from channel k, and $f_i^{(k)}$ represents the characteristic frequency of the i^{th} IMF in channel

k. These limitations stem from disregarding inter-channel correlations during decomposition, which becomes particularly problematic in SHM applications where distributed sensor networks capture closely spaced modal frequencies $\omega_{m}{}_{m=1}^{N}$ with $|\omega_i - \omega_j| < \epsilon$. The MEMD framework (presented as Algorithm 1 in Table 1) addresses these challenges through multivariate *sifting operations*.

2.2. Singular Spectrum Analysis (SSA)

Singular Spectrum Analysis (SSA) [2, 6] is a powerful signal decomposition technique that facilitates the analysis of time series data by breaking down the original signal into multiple interpretable components. This method allows for the identification of underlying trends, the mitigation of noise effects, and the reconstruction of a smoother signal, which can also aid in determining the order of the system. The SSA process consists of four key steps: embedding, singular value decomposition (SVD), grouping, and averaging. The algorithmic details for performing SSA are summarized in Algorithm 2 (Table 1), which outlines the mathematical procedures necessary for effective implementation.

Table 1: Algorithms for MEMD and SSA

Algorithm 1 MEMD

Require: Signals $\mathbf{x}_1(t) = \mathbf{x}_2(t) = \mathbf{x}(t)$, $i = 1$

Generate K randomly distributed directions θ_k

Compute projections:

$$b^{\theta_k}(t) = \mathbf{x}_1(t) \cdot (\mathbf{w}^{\theta_k})^T$$

Find local maxima $t_i^{\theta_k}$ of each projected signal $b^{\theta_k}(t)$

Interpolate maxima to get multivariate envelopes $\mathbf{e}^{\theta_k}(t)$

Compute mean envelope:

$$\mathbf{m}(t) = \frac{1}{K} \sum_{k=1}^{K} \mathbf{e}^{\theta_k}(t)$$

Extract the detail signal:

$$\mathbf{s}(t) = \mathbf{x}_1(t) - \mathbf{m}(t)$$

if $\mathbf{s}(t)$ is a Multivariate Intrinsic Mode Function (MIMF) **then**

  Set $\mathbf{d}_i(t) = \mathbf{s}(t)$, update i

else

  Update $\mathbf{x}_1(t) = \mathbf{s}(t)$ and repeat

end if

Update $\mathbf{x}_2(t) = \mathbf{x}_2(t) - \mathbf{d}_i(t)$

if $\mathbf{x}_2(t)$ lacks extrema **then**

  Stop; set trend $\mathbf{r}(t) = \mathbf{x}_2(t)$

end if

return MIMFs $\mathbf{d}_i(t)$ and residual $\mathbf{r}(t)$

Algorithm 2 SSA

Require: Time series $\mathbf{x} = \{x_1, x_2, ..., x_N\}$, window length L

Embedding: Construct the Hankel matrix:

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_L \\ x_2 & x_3 & \cdots & x_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-L+1} & x_{N-L+2} & \cdots & x_N \end{bmatrix}$$

SVD Decomposition:

$$\mathbf{X} = \sum_{i=1}^{L} \lambda_i \mathbf{u}_i \mathbf{v}_i^T$$

where $\mathbf{X}$ is the data matrix, λ_i are the singular values, $\mathbf{u}_i$ are the left singular vectors, and $\mathbf{v}_i$ are the right singular vectors from SVD.

Grouping: Select dominant components to extract signals

Reconstruction (Diagonal Averaging):

$$\tilde{x}_t = \sum_{i=1}^{r} \lambda_i u_{t,i} v_{t,i}$$

where $\tilde{x}_t$ represents the reconstructed signal at time t, λ_i are the singular values, $u_{t,i}$ are the left singular vectors at time t, and $v_{t,i}$ are the right singular vectors at time t, with r being the rank of the matrix.

The final decomposition consists of trend, periodic components, and noise.

return Reconstructed signal.

2.3. Logarithmic decrement for damping estimation

Damping in structures is quantified using the damping ratio, which can be estimated from vibration responses. This study utilizes the logarithmic decrement method, a widely recognized time-domain technique, valued for its simplicity and effectiveness in minimizing nonlinear effects [8]. Damping estimation is performed through the auto-correlation of the bridge response, defined by the auto-correlation function for a time-lag τ: $R_x(\tau) = E[x(t)x(t-\tau)]$. This function captures the decaying relationship between past and present values, facilitating damping estimation through peak identification. The logarithmic decrement method derives damping from successive peaks of R_x using $\delta = \ln \frac{x_i}{x_{i+n}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$, where δ is the logarithmic decrement and ζ is the damping ratio. From measured peaks, damping is determined as: $\zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$

3. PROPOSED METHODOLOGY

The proposed framework integrates MEMD and SSA to facilitate the extraction of meaningful features from multichannel time-series data. The process begins with MEMD, which decomposes the input response matrix X into a set of IMFs, each corresponding to distinct oscillatory modes embedded within the data. Subsequently, SSA is applied to refine these IMFs by exploiting SVD to group them based on their principal components. To provide a clear comprehension of the methodology, the key steps involved in the MEMD-SSA framework are outlined as follows:

- **STEP 1:** Input the data matrix (typically the output response) $\mathbf{X}$ of size $l \times n$ to the MEMD algorithm and collect the IMFs matrix $\mathbf{u}$ with dimensions (l, k, n), where l is the IMF length, k is the number of IMFs, and n is the number of channels.
- **STEP 2:** From $\mathbf{u}$, use the summation $\sum u(:, k-m : k, n)$, where m is the number of noisy IMFs separated by MEMD, and collect the remaining $(k-m)$ IMFs for SSA.This is performed for each channel separately.
- **STEP 3:** Apply SSA on the summation of the relevant IMFs for each channel n. First, perform SVD to extract eigenvalues, and then based on their magnitude, select the corresponding eigenvectors for reconstruction. Higher eigenvalues guide the selection of *significant components*.
- **STEP 4:** The final reconstructed signal is obtained by summing the selected principal eigenvectors from Step 3, thus getting the modal response of the input matrix $\mathbf{X}$.

4. APPLICATION TESTBED I - NUMERICAL SIMULATIONS

4.1. Preliminary investigations using a synthetic signal

For the purposes of investigating feasibility, efficacy, and applicability of the proposed approach, a synthetic signal is initially considered. The composite signal consists of three sine and cosine waveforms, each defined by distinct frequencies and amplitudes. The following context outlines the procedure employed to generate these signals (open-source MATLAB codes are provided as supplementary files):

- **Sampling Parameters:** The sampling frequency (f_s) is set to 500 Hz, and the signal duration (T) is 5 seconds. This resulted in a time vector t with a time step $\Delta t = \frac{1}{f_s}$, defined as: $t = 0 : \Delta t : T$ The frequencies of the waveforms are set to: $f_1 = 5\,\text{Hz}$, $f_2 = 20\,\text{Hz}$, $f_3 = 35\,\text{Hz}$.

- **Signal Construction:** The three simulated signals $y_1(t)$, $y_2(t)$, and $y_3(t)$ are generated as linear harmonic combinations with the corresponding amplitude coefficients $A_1 = 4$, $A_2 = 5$, $A_3 = 3$:

$$\begin{aligned} y_1(t) &= A_1 + A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) + A_3 \sin(2\pi f_3 t), \\ y_2(t) &= A_1 \sin(2\pi f_2 t) + A_2 \sin(2\pi f_1 t) + A_3 \sin(2\pi f_3 t), \\ y_3(t) &= A_3 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) + A_1 \sin(2\pi f_3 t). \end{aligned} \tag{2}$$

To simulate real-world conditions, Additive White Gaussian Noise (AWGN) is introduced to each signal, resulting in noisy signals with a Signal-to-Noise Ratio (SNR) set to 30 dB [9]. For clarity in visualization, the time-series representations of the signals are displayed up to 2 seconds, as extending them to 5 seconds would introduce unnecessary visual congestion. Fig.1(a) presents the simulated signals generated using Eq.2, along with their corresponding Fourier spectra. The outcomes of the proposed methodology, including the extracted components, are illustrated in Fig.2(a) and 2(b).

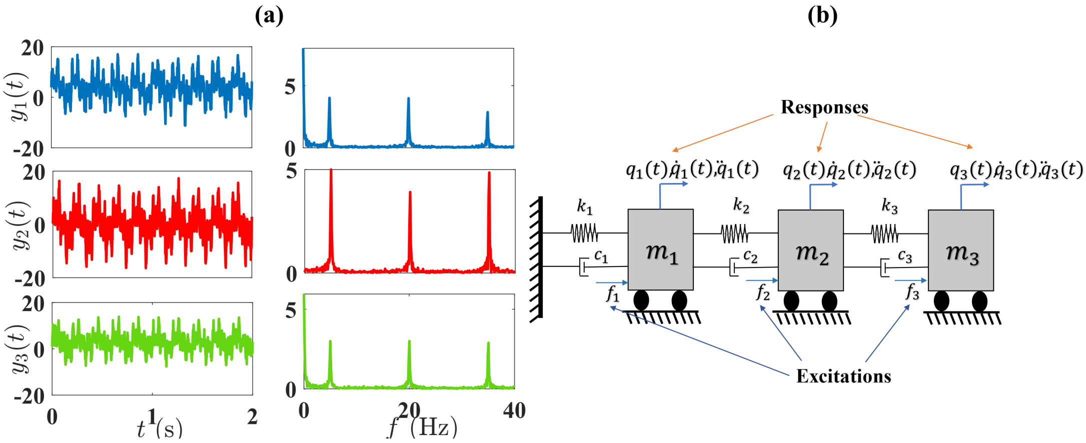

Figure 1: (a) Noise induced synthetic signals and their corresponding FFT, (b) Representation of a 3–DOF model.

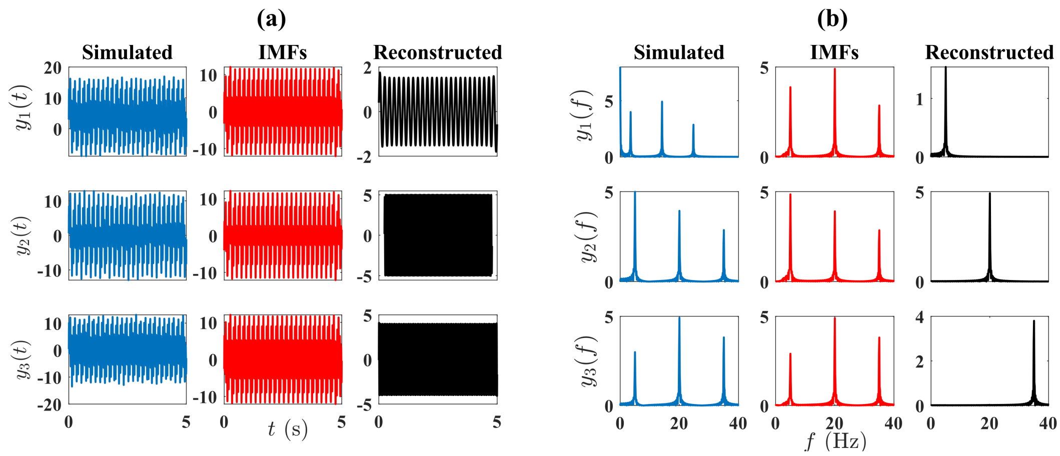

Figure 2: (a) FFT of responses, IMFs and reconstructed signal, (b) Responses and reconstructed signal after application of MEMD+SSA.

4.2. Secondary applications using a dynamic 3–DOF model

The governing equation of motion for a three-degree-of-freedom (3-DOF) system can be expressed as:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{F}(t), \tag{3}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, $\ddot{\mathbf{q}}(t)$, $\dot{\mathbf{q}}(t)$, $\mathbf{q}(t)$ is the acceleration, velocity and displacement matrix respectively, $\mathbf{F}(t)$ is the external force vector (considered as Gaussian white noise of zero-mean and unit standard deviation). The natural frequencies ω_n of the system can be determined by solving the eigenvalue problem. The damping ratio ζ for each mode is given by: $\zeta_i = \frac{c_i}{2\sqrt{k_i m_i}}$, where c_i, k_i, and m_i correspond to the modal damping, stiffness, and mass of the i^{th} mode, respectively. The system matrices of the 3-DOF system – illustrated in Fig.1(b) – are considered on the basis of the damping ratios for each mode at $\zeta_1 = 0.12$, $\zeta_2 = 0.05$, and $\zeta_3 = 0.03$:

$$\mathbf{M} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0.4111 & -0.1000 & 0 \\ -0.1000 & 0.1389 & -0.0389 \\ 0 & -0.0389 & 0.0389 \end{bmatrix}$$

Fig.3(a) presents the response of the 3-DOF model alongside its FFT, illustrating the presence of closely spaced modal frequencies – a characteristic that poses challenges for traditional modal identification techniques. The application of MEMD to this dataset reveals mode-mixing, as observed in Fig.3(a), where the IMFs exhibit spectral contamination due to overlapping frequency components. The extracted frequency values and corresponding damping ratios – summarized in Table2 – further emphasize the complexities arising from spectral interference.

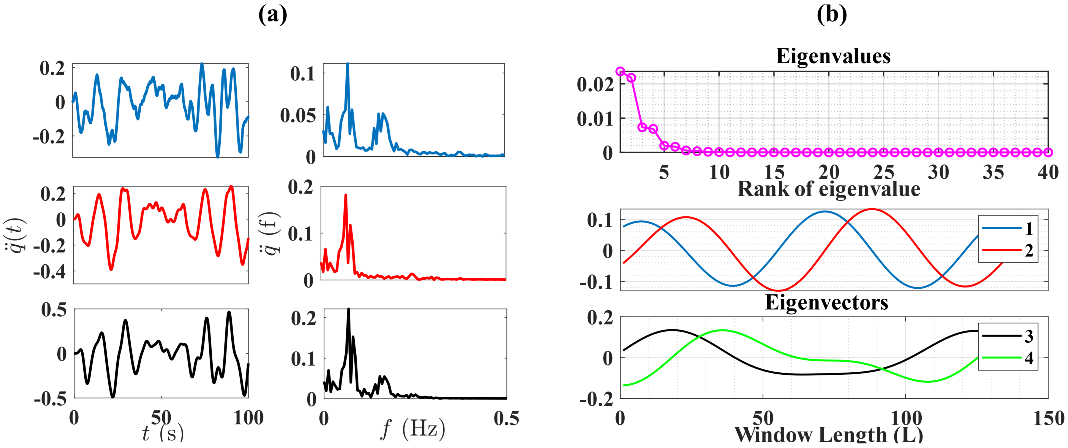

Figure 3: (a) Responses and FFT from 3–DOF model, (b) Eigenvalues and eigenvectors from SVD.

To mitigate these effects, SSA is employed to facilitate the selection of dominant modal components while effectively suppressing noise. The eigenvalues obtained from SVD, as shown in Fig.3(b), serve as selection criteria for the reconstruction process, ensuring that only the most significant spectral contributions are retained. Drawing from the eigenvalue distribution in Fig.3(b), an attenuating eigenvalue of **4** was selected, optimizing the trade-off between mode separation and signal fidelity. *This choice aligns with the broader framework of data-driven modal analysis, wherein adaptive decomposition strategies –* like the proposed **MEMD+SSA** approach – *enhance the robustness of frequency extraction in the presence of measurement noise and mode-mixing effects*. By selectively retaining dominant eigenvectors, the reconstructed response in Fig.4(a) demonstrates improved spectral coherence, thereby enabling more accurate estimation of modal parameters. *The application of SSA ensures that only physically meaningful modes contribute to the final reconstruction, reducing spectral leakage and enhancing signal fidelity.*

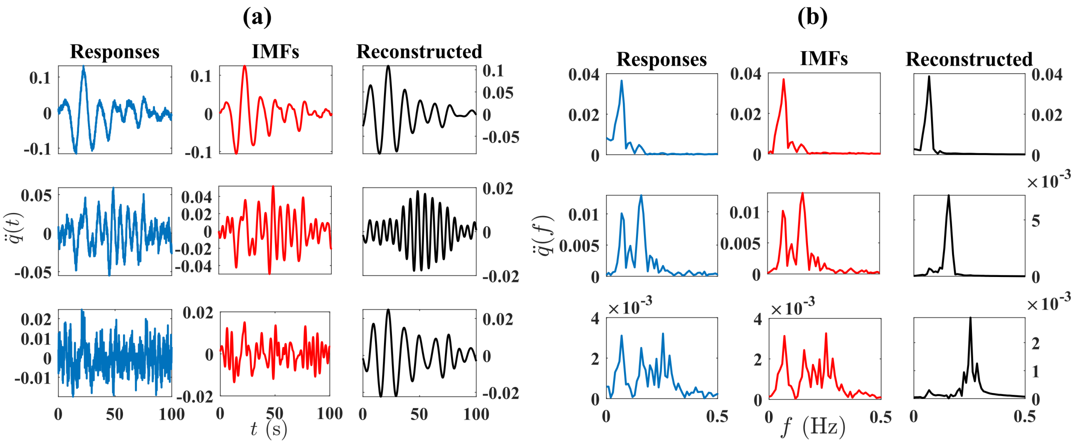

Figure 4: (a) Responses and reconstructed signal for the 3DOF systems after applying MEMD+SSA, (b) FFT of responses, IMFs and the reconstructed signal.

5. APPLICATION TESTBED II – CONSIDERATIONS FOR THE LYSEFJORD BRIDGE

The Lysefjord Bridge dataset [7] consists of dynamic response measurements obtained from multiple sensors deployed across the structure. The presence of closely spaced modes, as depicted in Fig.5(a), complicates traditional modal analysis, where small perturbations in structural conditions can induce subtle shifts in modal frequencies. To address this, MEMD is employed to decompose the structural response into IMFs, while SSA facilitates the separation of dominant modal components from noise, thereby improving feature extraction.

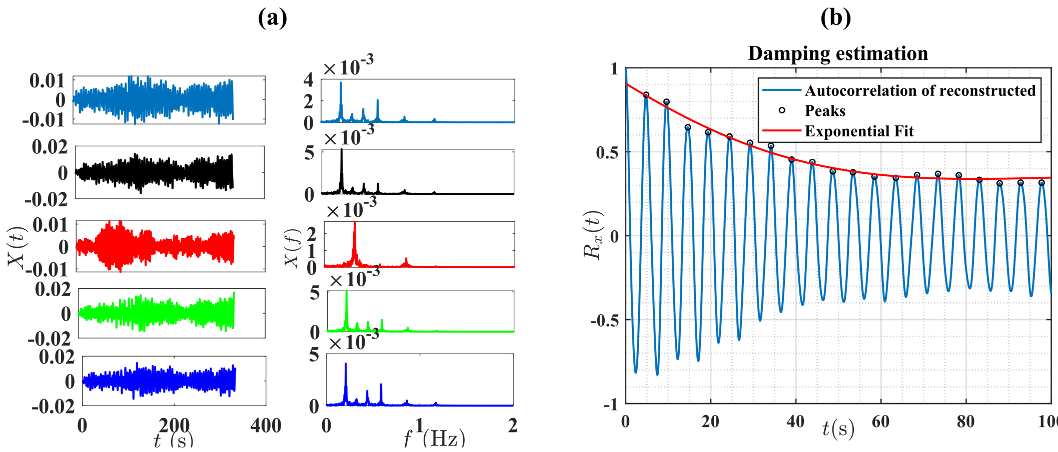

Figure 5: (a) Responses and FFTs of the Lysefjord Bridge, (b) Damping estimation from reconstructed signals after MEMD+SSA for the Lysefjord bridge.

Fig.5(a) presents the bridge responses alongside their FFT, illustrating the modal complexity. Damping estimation, conducted via the logarithmic decrement method (Fig.5(b)), provides insight into energy dissipation, with corresponding values detailed in Table2. Fig.6(a) and 6(b) display the refined responses following the application of MEMD-SSA. The results confirm that the proposed approach effectively resolves closely spaced modes, ensuring accurate identification of modal parameters.

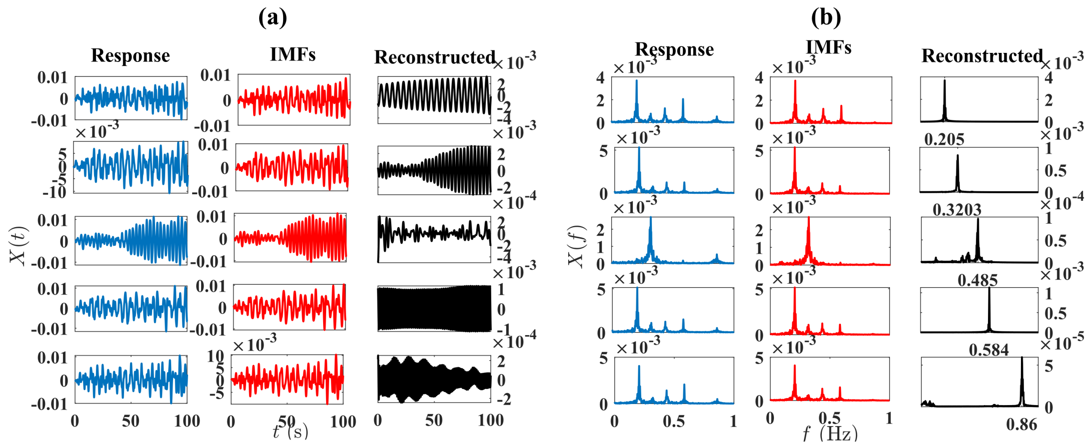

Figure 6: (a) Responses, IMFs and reconstructed signals, (b) FFT of responses, IMFs and reconstructed signal.

6. CONCLUSIONS

The proposed MEMD-SSA framework demonstrates a significant improvement in modal decomposition by systematically addressing the challenges posed by closely spaced frequencies, mode-mixing, and measurement noise. The numerical investigations on a 3-DOF system reveal that traditional MEMD, while effective in decomposing multichannel signals, suffers from *spectral leakage* and *mode entanglement*, which SSA effectively mitigates through eigenvalue-based filtering. The validation using the Lysefjord

Bridge dataset further establishes the robustness of the approach, as SSA successfully isolates dominant modes, preserving their physical significance while suppressing noise artifacts. The damping estimation results – derived via logarithmic decrement – exhibit consistency with theoretical expectations. Additionally, the observed correlation between eigenvalue attenuation and modal significance highlights the importance of optimal eigenvalue selection in refining reconstructed signals. These findings position MEMD-SSA as a computationally effective and structurally meaningful tool for high-fidelity modal analysis in SHM applications. While this method offers several advantages, the authors suggest future research explorations to address the following limitations: **High Computational Cost** – Signal reconstruction time grows exponentially with length, demanding heavy processing. **Mode Alignment vs. IMF Quality** – MEMD aligns modes across channels but may still suffer from mode mixing, affecting IMF separation.

Table 2: Damping ratio (ζ) and frequency (f(Hz)) values for the 3DOF system and Lysefjord Bridge.

Mode	**3–DOF**				**Lysefjord Bridge**			
	ζ (ratio)		f (Hz)		ζ (ratio)		f (Hz)	
	Original	Estimated	Original	Estimated	Original	Estimated	Original	Estimated
1	0.12	0.1134	0.067	0.0685	0.05	0.047	0.2046	0.2050
2	0.05	0.048	0.1592	0.1566	0.05	0.05	0.3189	0.3203
3	0.03	0.036	0.2674	0.2544	0.05	0.053	0.4391	0.4851
4	...	...	...	...	0.05	0.05	0.5852	0.5840
5	...	...	...	...	0.05	0.046	0.8643	0.8603

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REFERENCES

[1] SW. Doebling, CR. Farrar, MB. Prime, and DW. Shevitz. Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review. 1996.

[2] B. Bhowmik, M. Krishnan, B. Hazra, and V. Pakrashi. Real-time unified single-and multi-channel structural damage detection using recursive singular spectrum analysis. *Structural Health Monitoring*, 18(2):563–589, 2019.

[3] A. Sharma and B. Bhowmik. When fire attacks: How does concrete stand up to heat? *arXiv preprint arXiv:2408.15756*, 2024.

[4] Lakhadive Mehulkumar R, Anshu Sharma, and Basuraj Bhowmik. Investigating dimensionally-reduced highly-damped systems with multivariate variational mode decomposition: An experimental approach. *arXiv preprint arXiv:2503.11689*, 2025.

[5] N. ur Rehman, C. Park, N.E. Huang, and D.P. Mandic. Emd via memd: multivariate noise-aided computation of standard emd. *Advances in adaptive data analysis*, 5(02):1350007, 2013.

[6] H. Hassani. Singular spectrum analysis: methodology and comparison. 2007.

[7] E. Cheynet, JB. Jakobsen, and J. Snæbjörnsson. Buffeting response of a suspension bridge in complex terrain. *Engineering Structures*, 128:474–487, 2016.

[8] F. Huang, X. Wang, Z. Chen, X. He, and Y. Ni. A new approach to identification of structural damping ratios. *Journal of Sound and Vibration*, 303(1-2):144–153, 2007.

[9] D.R. Pauluzzi and N.C. Beaulieu. A comparison of snr estimation techniques for the awgn channel. *IEEE Transactions on communications*, 48(10):1681–1691, 2000.