11th International Operational Modal Analysis Conference
20 - 23 May 2025 | Rennes, France

Examining environmental influences on natural frequencies with the help of local distance correlation

Carina Beering

Helmut Schmidt University Hamburg, beeringc@hsu-hh.de

ABSTRACT
In order to investigate the influence of environmental effects on natural frequencies, we use local distance correlation (LDC) to accommodate the nature of environmental data, that is local stationarity. LDC pattern help to identify similarities and differences of these influential variables. In addition to that, the examination of LDC pattern between frequencies of both the same and different mode types allow to compare their temporal progress. The study uses data from a monitoring system installed on the steel railway bridge KW51 in Leuven in Belgium.

Keywords: Natural frequencies, environmental influences, local stationarity, local distance correlation

1. INTRODUCTION

When tracking natural frequencies of a structure, it is important to keep the environmental effects in mind. Changes in temperature, for example, have an effect as well as other natural conditions. To quantify these effects or to find similarities in their respective behavior, statistical methods come into play. A classical approach is to observe the correlation. However, this procedure only captures linear dependencies and is blind for more complex dependence structures like quadratic dependencies as an example. The concept of distance correlation as introduced by [1] provides a sound way out since it is not only not restricted to linear dependence, but is also able to detect independence and does not require normality. Another particularity to consider is the fact that most sensor data is not stationary. In view of temperature, we have seasonal effects and daily pattern implying changes in mean and variance. Nevertheless, looking at small time frames, we observe stationary behavior. Thus, the data can be considered as locally stationary. Therefore, we use the extension of [2] of the original version of distance correlation, the LDC, to investigate environmental effects. In addition to that, we examine local distance correlation pattern between frequencies of the same as well as of different mode types to find similarities and differences in their temporal progress. Our study is based on data provided by [3] originating from the steel railway bridge KW51 in Leuven in Belgium.

The paper is structured as follows: In Section 2, we provide a more detailed description of the LDC and its components as well as their empirical counterparts. Section 3 contains the study applying the statistical concepts of the previous section. Lastly, Section 4 concludes with a roundup.

2. LOCAL DISTANCE CORRELATION

Distance correlation in its original for as introduced by [1] dealt with independent and identically distributed data. Based thereupon, [4] proposed a version for stationary data, which, in turn, inspired [2] to present a modification suitable for local stationary time series, the LDC. This modifications answers our needs regarding SHM data. Consequently, we consider two locally stationary processes $(Y_{t,T})_{t=1}^{T}$ and $(Z_{t,T})_{t=1}^{T}$ for $T \in \mathbb{N}$ as described by [5] satisfying the assumptions of [2]. Thus, the respective strictly stationary companion processes $\left(\widetilde{Y}_t(u)\right)_{t\in\mathbb{Z}}$ and $\left(\widetilde{Z}_t(u)\right)_{t\in\mathbb{Z}}$ for a time point $u \in [0,1]$ approximate the original processes locally, that is in a neighborhood of u. For the sake of notational simplicity, we concentrate on the one-dimensional case and the contemporaneous contemplation of the two processes. A more flexible approach in the time domain is described in [6]. As the concept of distance correlation, and hence LDC as well, is based on characteristic functions belonging to the companion processes, consider for $s_1, s_2 \in \mathbb{R}$

$$\varphi_Y(u; s_1) := Ee^{is_1\widetilde{Y}_0(u)} \quad \text{and} \quad \varphi_Z(u; s_2) := Ee^{is_2\widetilde{Z}_0(u)} \tag{1}$$

plus the joint characteristic function

$$\varphi_{Y,Z}(u; s_1, s_2) := Ee^{is_1\widetilde{Y}_0(u)+is_2\widetilde{Z}_0(u)}. \tag{2}$$

In addition to the characteristic functions, we need a weight function w to form the LDC, which we borrow from [2], that is

$$w(s_1, s_2) = (\pi s_1 s_2)^{-2} \tag{3}$$

for $s_1, s_2 \in \mathbb{R} \setminus \{0\}$. That way, the weight function is always positive. Then, the non-lagged local distance covariance as in [2] states as follows:

$$\mathcal{V}^2_{Y,Z}(u) = \int_{\mathbb{R}\times\mathbb{R}} |\varphi_{Y,Z}(u; s_1, s_2) - \varphi_Y(u; s_1)\varphi_Z(u; s_2)|^2 w(s_1, s_2)\, ds_1 ds_2. \tag{4}$$

With $\mathcal{V}^2_{Y,Z}(u)$, the LDC is defined as

$$\mathcal{R}^2_{Y,Z}(u) = \begin{cases} \frac{\mathcal{V}^2_{Y,Z}(u)}{\sqrt{\mathcal{V}^2_{Y,Y}(u)\mathcal{V}^2_{Z}(u)}}, & \mathcal{V}^2_{Y,Y}(u)\mathcal{V}^2_{Z,Z}(u) > 0, \\ 0, & \mathcal{V}^2_{Y,Y}(u)\mathcal{V}^2_{Z,Z}(u) = 0. \end{cases} \tag{5}$$

Relying on the characteristic functions of the unobservable companion processes, both Eq. (4) and Eq. (5) need an empirical counterpart to be used with data. Aligning with the definitions made before, we adopt once again the ones of [2]. To do so, we begin by introducing the so-called local sample analogues of the characteristic functions used in Eq. (4) and Eq. (5). This becomes necessary since the classical estimators cannot be used in consequence of the SHM data's local stationarity. Hence, consider

$$\widehat{\varphi}_Y(u; s_1) := \frac{1}{b_T T}\sum_{t=1}^{T} K\left(\frac{t/T - u}{b_T}\right) e^{is_1 Y_{t,T}} \tag{6}$$

and

$$\widehat{\varphi}_Z\left(u; s_2\right) := \frac{1}{b_T T} \sum_{t=1}^{T} K\left(\frac{t/T - u}{b_T}\right) e^{i s_2 Z_{t,T}} \tag{7}$$

along with the joint version

$$\widehat{\varphi}_{Y,Z}\left(u; s_1, s_2\right) := \frac{1}{b_T T} \sum_{t=1}^{T} K\left(\frac{t/T - u}{b_T}\right) e^{i s_1 Y_{t,T} + i s_2 Z_{t,T}}. \tag{8}$$

The locality is achieved by using a Kernel function K combined with a bandwidth b_T depending on the sample size T. For detailed regularity conditions with respect to both K and b_T, see [2]. Re-using the weight function w from Eq. (3), the empirical local distance covariance reads as follows:

$$\widehat{\mathcal{V}}^2_{Y,Z}(u) = \int_{\mathbb{R}\times\mathbb{R}} \left|\kappa_T \widehat{\varphi}_{Y,Z}\left(u; s_1, s_2\right) - \widehat{\varphi}_Y\left(u; s_1\right) \widehat{\varphi}_Z\left(u; s_2\right)\right|^2 w\left(s_1, s_2\right)\, ds_1\, ds_2. \tag{9}$$

Comparing Eq:4 and Eq. 9, we note the additional factor κ_T. More precisely, κ_T is of the following form:

$$\kappa_T := \frac{1}{b_T T} \sum_{t=1}^{T} K\left(\frac{t/T - u}{b_T}\right). \tag{10}$$

Once more, we refer to [2] for a rationale on the requisiteness of κ_T. Lastly, the empirical LDC is obtained by

$$\widehat{\mathcal{R}}^2_{Y,Z}(u) = \begin{cases} \frac{\widehat{\mathcal{V}}^2_{Y,Z}(u)}{\sqrt{\widehat{\mathcal{V}}^2_{Y,Y}(u)\widehat{\mathcal{V}}^2_{Z,Z}(u)}}, & \widehat{\mathcal{V}}^2_{Y,Y}(u)\widehat{\mathcal{V}}^2_{Z,Z}(u) > 0, \\ 0, & \widehat{\mathcal{V}}^2_{Y,Y}(u)\widehat{\mathcal{V}}^2_{Z,Z}(u) = 0. \end{cases} \tag{11}$$

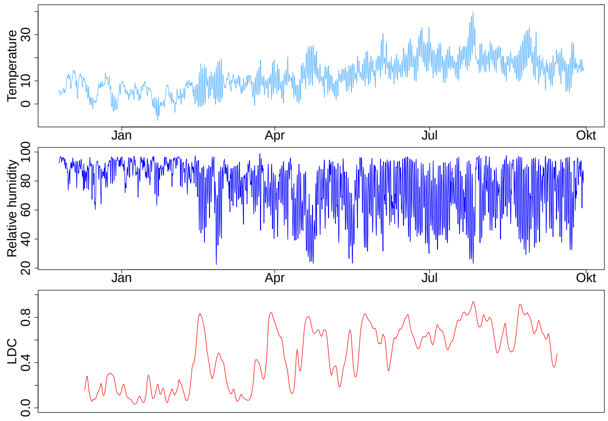
Consistency of these estimators is given by Theorem 4.2 in [2], which justifies their use in our study in Section 3..

3. ENVIRONMENTAL PHENOMENA AND NATURAL FREQUENCIES

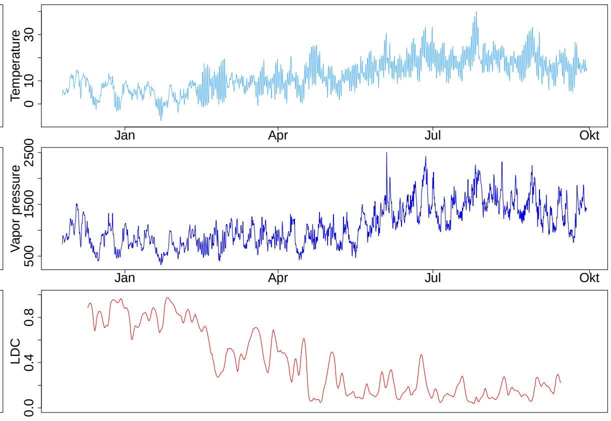
In this section, the concept of LDC is used to study environmental phenomena and the evolution of natural frequencies of different mode types over time. The study is based on data resulting from the monitoring of the KW51 bridge in Leuven in Belgium, see [3] for further information of the data generation. In addition to the sensors mounted directly on the steel railway bridge, a weather station located at the Vliet Building from the KU Leuven Building Physics Section with an approximate distance of five kilometers bridge complemented the data set with environmental data. To compute the local sample analogues of the different characteristic functions, an Epanechnikov kernel was used. Subsection 3.1 begins with the examination of said environmental data, whereas Subsection 3.2 puts different mode types in comparison with each other and, on the back of this, with the bridge temperature.

3.1. Local distance correlation of weather data

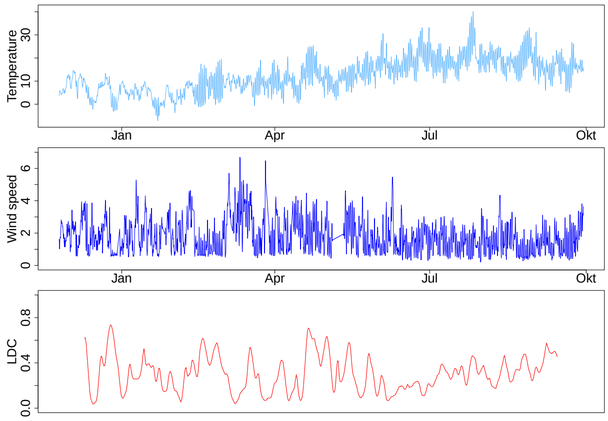
The weather station at the Vliet Building collects data for several environmental factors, among which are temperature, relative humidity, vapor pressure and wind speed. To begin with, we examine the LDC of the temperature combined with all three other factors, respectively. The results are displayed Subfigures 1a, 1b and 1c. In the first row of each combined plot, we see the progress of temperature, whereas in the second row, the other factors are shown. The last row contains the respective LDC. Starting with the first graphic, that is Subfigure 1a, we notice a rise in variability over time of both



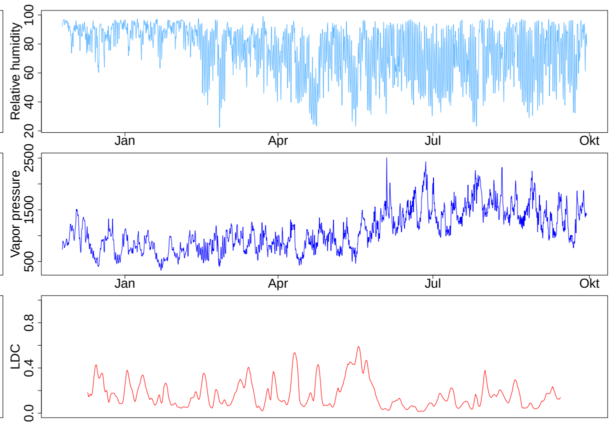
(a) Temperature and relative humidity



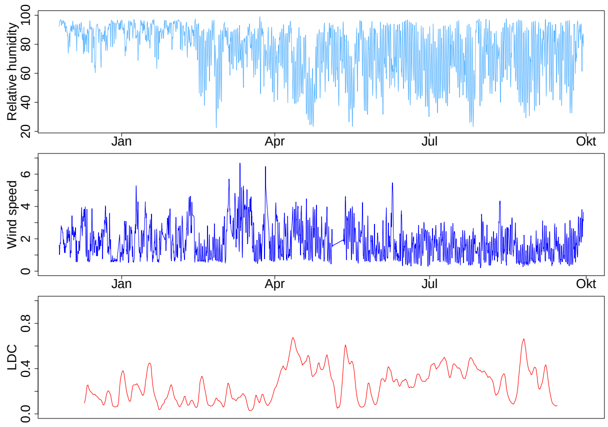
(b) Temperature and vapor pressure



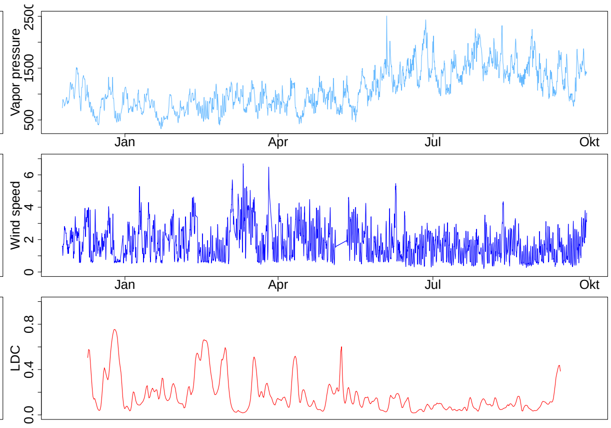
(c) Temperature and wind speed



(d) Relative humidity and vapor pressure



(e) Relative humidity and wind speed



(f) Vapor pressure and wind speed

Figure 1: Data from different weather phenomena and corresponding LDC by pairs.

temperature and relative humidity. However, this leads only to an upward trend in the LDC, but not to a distinctive pattern. Moving on to Subfigure 1b, the LDC shows the opposite behavior, although colder temperatures, in general, go along with a lower vapor pressure and higher temperatures with higher vapor pressure. Thus, the similarity of the basic trends results in a LDC greater than zero, which would imply independence of the underlying time series, but does not imply a deeper coherence. In the following Subfigure 1c, the LDC belonging to the pair of temperature and wind speed fluctuates, but remains on a comparably low level. Right next to it in Subfigure 1d, the LDC of relative humidity and vapor pressure behaves in a similar way, that is unsteady, but even lower. The third row of Figure 1, displaying the remaining pairs in Subfigures 1e and 1f, get in line with the previous one. All in all, regarding these six environmental factors, there is no specific pattern in the LDC allowing us to exclude a certain factor from the analysis. However, a counterexample can be found in [6], where the LDC of air and bridge temperature are considered.

3.2. Local distance correlation and the evolution of natural frequencies

In this subsection, we examine the LDC of natural frequencies appurtenant to different modes. Beginning with modes of the same type, we consider two global vertical modes of the KW51 bridge. In [3], ambient vibration data has been used to perform an operational modal analysis resulting in the evolution of modal parameters over time. We focus on the identified natural frequencies, beginning with modes 12 and 13. They are both global vertical modes. Figure 2 shows the respective natural frequencies for the month of January in 2019. The evolution is quite stable with limited variation until around the 19th of January. For roughly a week, the natural frequencies rise strongly in comparison to the time before. Moreover, the variability increases as well. Looking at the third row of Figure 2, we see a steep rise of the LDC at the same time, apart from some negative peaks.

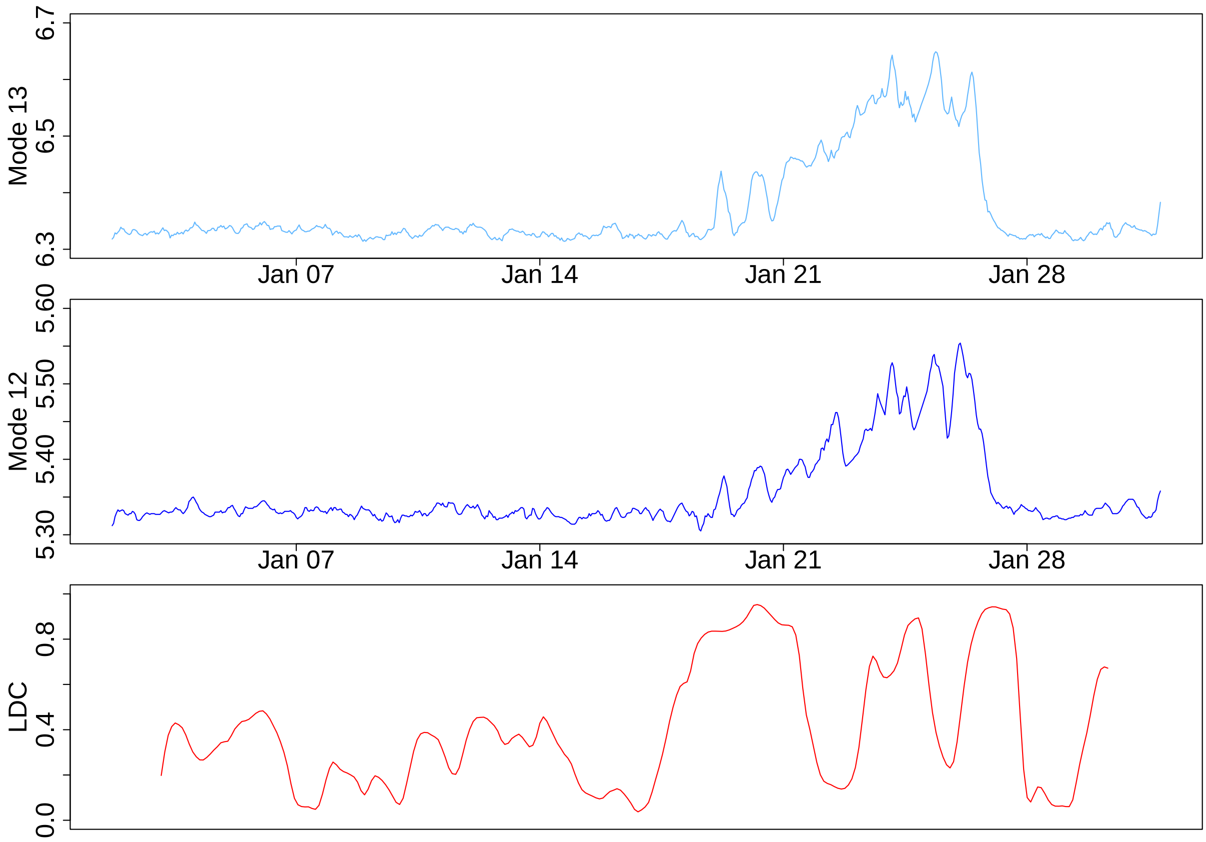


Figure 2: Evaluation of natural frequencies of modes 12 and 13 in combination with their LDC over time.

Exchanging mode 12 for mode 3, a lateral mode of the bridge deck, we observe the same behavior of the natural frequencies and the LDC, as displayed in Figure 3. However, in the beginning of January, the LDC is, disregarding some peak, in general a bit lower than in the previous case. In conclusion, the LDC for either the same or different mode types is rather low, yet it detects external factors influencing the evolution of the frequencies.

To tie in with the previous subsection, we consider the bridge temperature in tandem with the natural frequencies belonging to mode 13. The use of the bridge temperature in lieu of the temperature measured by the weather station allows for a contemporaneous study since we do not have to account for the time the bridge needs to adjust to a change in temperature. Figure 4 shows temperatures below zero for the week of the rise in mode 13's natural frequency. Nevertheless, the LDC displayed in the last row

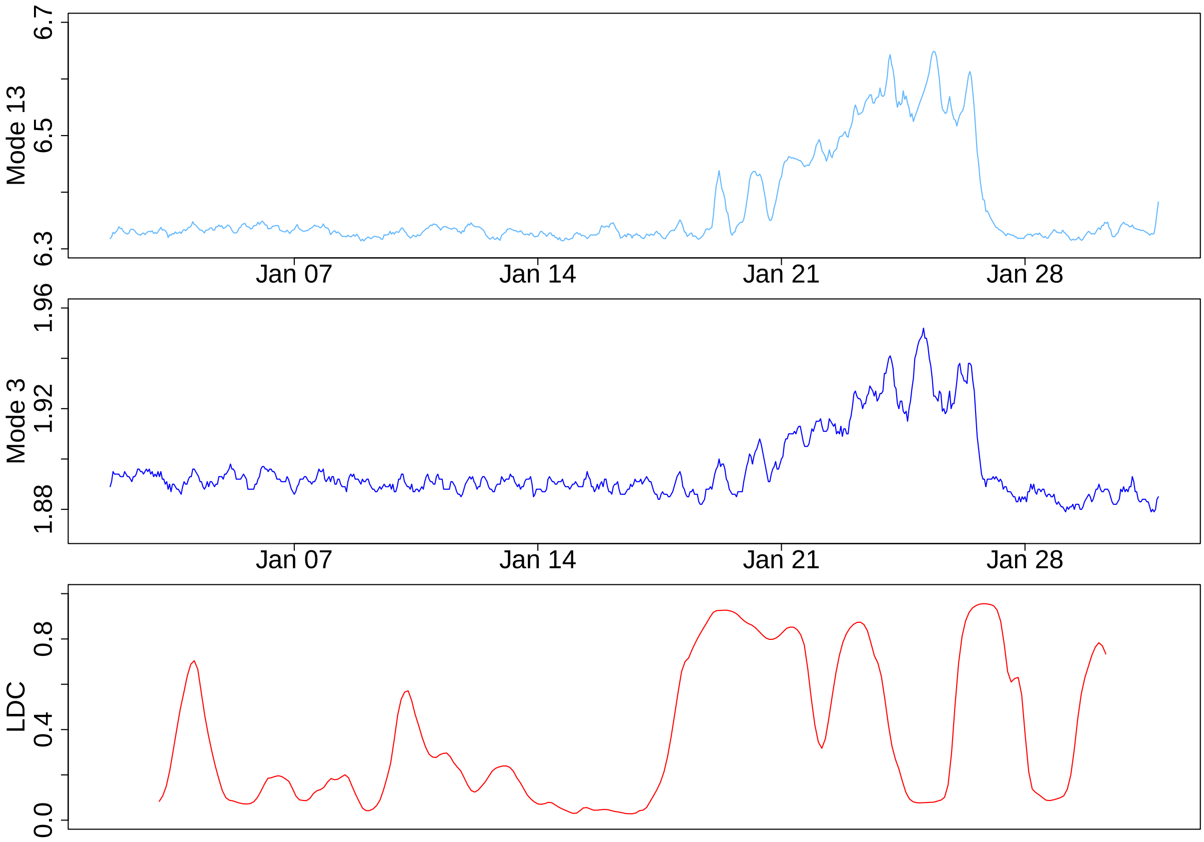


Figure 3: Evaluation of natural frequencies of modes 3 and 13 in combination with their LDC over time.

does not explain the connection to its full extend. This falls in line with the findings of Subsection 3.1. One single weather phenomenon is not enough to expound the change seen in the natural frequency's comportment, but acts as a starting point of further investigation by combining data stemming from different phenomena.

4. CONCLUSIONS AND OUTLOOK

Local distance correlation is a powerful tool to accommodate the stochastic needs of environmental and structural health data. It is able to find systematic similarities in time series or the lack thereof. Moreover, due to its versatile applicability, it opens the way to interdisciplinary cooperation, especially in the field of structural health monitoring.

A next step in the analysis of environmental influences on the evolution of natural frequencies would be the combination of different types of weather data in order to obtain a wider explanation of the variability of results from the operational modal analysis for the natural frequencies. Furthermore, the consideration of modes of the same type can help to substantiate empirical findings.

ACKNOWLEDGMENTS

This research is funded by dtec.bw - Digitalization and Technology Research Center of the Bundeswehr. dtec.bw is funded by the European Union - NextGenerationEU.

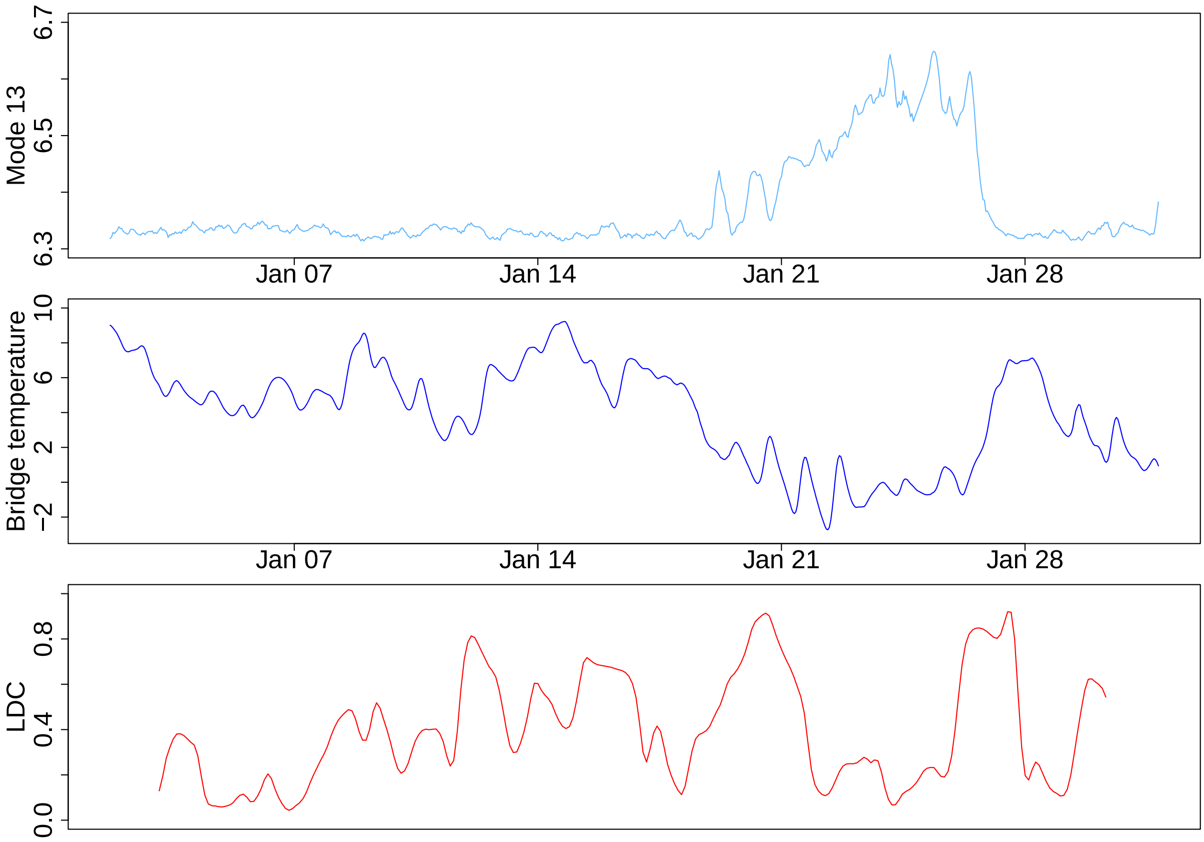


Figure 4: Evaluation of the natural frequency of mode 13 paired with the bridge temperature together with their time-respective LDC.

REFERENCES

[1] G.J. Székely, M.L. Rizzo, and N.K. Bakirov. Measuring and testing dependence by correlation of distances. *The Annals of Statistics*, 35:2769–2794, 2007.

[2] C. Jentsch, A. Leucht, M. Meyer, and C. Beering. Empirical characteristic functions-based estimation and distance correlation for locally stationary processes. *Journal of Time Series Analysis*, 41: 110–133, 2020.

[3] K. Maes and G. Lombaert. Monitoring railway bridge KW51 before, during, and after retrofitting. *Journal of Bridge Engineering*, 23(3), 20217.

[4] R.A. Davis, M. Matsui, T. Mikosch, and P. Wan. Applications of distance correlation to time series. *Stochastic Processes and their Applications*, 1191:3699–37183, 2018.

[5] R. Dahlhaus. Fitting time series models to nonstationary processes. *The Annals of Statistics*, 25: 1–37, 1997.

[6] C. Beering. Bridge monitoring with the help of local distance correlation. *e-Journal of Nondestructive Testing*, 2024.